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Superstring standard model from $\mathbf{Z}_{12-I}$ orbifold compactification with and without exotics, and effective R-parity

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Abstract: We construct a supersymmetric standard model in the context of the $\mathbf{Z}_{12-I}$ orbifold compactification of the heterotic string theory. The gauge group is $\mathrm{SU(3)_c} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y} \times \mathrm{U(1)}^4 \times [\mathrm{SO(10)} \times \mathrm{U(1)}^3]'$. We obtain three chiral families, $3 \times \{Q, d^c, u^c, L, e^c, \nu^c\}$, and Higgs doublets. There are numerous neutral singlets many of which can have VEVs so that low energy phenomenology on Yukawa couplings can be satisfied. In one assignment (Model E) of the electroweak hypercharge, we obtain the string scale value of $\sin^2\theta_W^0 = \frac{3}{8}$ and another exactly massless *exphoton* (in addition to the photon) coupling to exotic particles only. There are color triplet and anti-triplet exotics, α and $\overline{\alpha}$, $\mathrm{SU(2)_L}$ doublet exotics, δ and $\overline{\delta}$, and $\mathrm{SU(3)_c} \times \mathrm{SU(2)_L}$ singlet but $Y = \frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}, \frac{1}{3}$ exotics, $\xi, \eta, \overline{\xi}, \overline{\eta}$. We show that all these vector-like exotics achieve heavy masses by appropriate VEVs of neutral singlets. One can find an effective R-parity between light (electroweak scale) particles so that proton and the LSP can live sufficiently long. In another assignment (Model S) of the electroweak hypercharge, there does not appear any exotic particle but $\sin^2\theta_W^0 = \frac{3}{14}$.

Keywords: Compactification and String Models, Supersymmetric Standard Model, Quark Masses and SM Parameters.

Contents

1. Introduction

There has been numerous attempts to obtain supersymmetric standard models (SSM) from the orbifold compactification of heterotic string [1−3]. In the old standardlike models, the attempts were just obtaining the standard model gauge group and three families [4, 5]. In a recent past, more ambitious attempts such as $\sin^2\theta_W = \frac{3}{8}$ [6], one pair of Higgs doublets [7], and neutrino masses [8] were tried to be explained. More recently, the Yukawa coupling structure has been looked for [9−11]. Among these, in particular we find the GUT model of [11] is satisfactory for the strong CP solution via the QCD axion although a GUT scale axion decay constant is needed [12], and for the approximate R-parity violation [13].

From the proton longevity problem, the R-parity or matter parity must be exact or feebly violated if it is an approximate one [14]. Otherwise, the string model construction must be treated as an academic exercise. Even with a successful R-parity, still there may be a good deal of phenomenological problems to be overcome. Successful Yukawa coupling structure is the next immediate concern in particle phenomenology. It is known that the Yukawa coupling structure can be satisfied with the help of numerous singlets [10, 11]. The next important concern is the vacuum stabilization problem or the problem of flat directions. But the vacuum stabilization problem is the most difficult one to analyze. At present, we are not yet at the stage to deal with this flat direction problem and we defer this flat direction problem until we find a model satisfying other phenomenological constraints. The approximate R-parity of [13] is the result of GUT scale VEVs of $\mathbf{10}^H$ and $\overline{\mathbf{10}}^H$ in the flipped SU(5) model. This hints that it may be possible to obtain an exact R-parity if one succeeds in obtaining an SSM without such constraint on the GUT scale VEVs.[1] Since the SSM through the flipped SU(5) was obtained from a $\mathbf{Z}_{12-I}$ compactification, we look for a SSM directly in the $\mathbf{Z}_{12-I}$ compactification. If found, the model is free from the constraints of $\mathbf{10}^H$ and $\overline{\mathbf{10}}^H$ in the flipped SU(5) model. But, then in a direct SSM construction one must check the doublet-triplet splitting more carefully. A computer search of SSMs is in principle possible but it is very difficult to put in all the phenomenological requirements. At some stage a model by model study is necessary. For example, we encounter a difficulty of calculating the determinant of mass matrix of singlet exotics in models with exotics whose number is much more than 10. The determinant being zero up to some order of Yukawa couplings does not necessarily mean that exotics do not obtain mass since still higher orders might render a non-vanishing determinant. Fortunately, for the $\mathbf{Z}_{12-I}$ compactification toward a direct SSM, it has been possible to find out an SSM without the computer search.

In this paper, we present an SSM in the $\mathbf{Z}_{12-I}$ compactification which can allow an *exact R-parity for low energy* (electroweak scale) fields, which will be called an *effective R-parity.* In the full theory, the R-parity is not exact but the violation occurs through the type, $(heavy\ field) \to (light\ fields)$. With this kind of effective R-parity, still the lightest supersymmetric particle (LSP) can be a stable CDM candidate.

[1] If unlucky, such constraints will be replaced by GUT scale constraints on singlet VEVs, which has to be checked carefully.

The R-parity in the SO(10) GUT is achieved by different assignments of quarks and leptons and Higgs doublets: in the the spinor **16** for quarks and leptons and the vector **10** for Higgs doublets. This kind of spinor-vector disparity can be adopted in the *untwisted sector* of heterotic string also. Let us consider only the E_8 part of the heterotic string [15] for an illustration. The *untwisted sector* massless matter spectrum in E_8 can be $P^2 = 2$ weights distinguished by the spinor or the vector properties

$$\mathcal{S} : ([+ + + + + + + +]) \quad \mathcal{V} : (\underline{\pm 1 \ \pm 1\ 0\ 0\ 0\ 0\ 0\ 0})$$

where $\pm$ represents $\pm\frac{1}{2}$, the notation [ ] means including even number of sign flips inside the bracket, and the underline means permutations of the entries on the underline. It is obvious that cubic Yukawa couplings constructed with $\mathcal{S}$ and $\mathcal{V}$ respect a $\mathbf{Z}_2$ parity. But including matter from the twisted sector, the study is more complex and we need the full machinery of Yukawa couplings, including nonrenormalizable terms. Here, the inclusion of neutral singlets, among which some needed singlet VEVs can take the $\langle\mathcal{S}\rangle$ form, spoils this idea of an exact R-parity. This needed singlet $\langle\mathcal{S}\rangle$ is the reason that exact R-parity models are very rare if not impossible. It is closely linked to the assignment of the electroweak hypercharge Y. We will show two interesting Y assignments with the resulting physics such as exotics, $\sin^2\theta_W$ and R-parity.

For the R-parity to be exact, it must be a subgroup of an anomaly-free U(1) gauge group, i.e. it must be a discrete gauge symmetry [16], otherwise large gravitational corrections such as through wormhole processes may violate it. Finding an anomaly free U(1) gauge symmetry direction whose $\mathbf{Z}_2$ subgroup is an R-parity is necessary for this purpose. For the U(1) gauge symmetry toward the R-parity, we use $U(1)_\Gamma$. For the study of some Yukawa couplings, another $\mathrm{U}(1)_{\Gamma'}$ symmetry is more convenient. When we start to list the massless states, we include these U(1) charges , Γ and Γ', even before presenting their definitions.

In section 2, we present an SSM from a $\mathbf{Z}_{12-I}$ compactification. Sections 3–5 discuss Model E. In section 3, we list exotic states which form vectorlike representations. We show how these exotics obtain masses by VEVs of neutral singlets. In section 4, we discuss that there exist D- and F-flat directions. In section 5, we find a U(1) direction whose $\mathbf{Z}_2$ subgroup can be used as an effective R-parity in Model E. In section 6, we discuss Model S. The arguments on D- and F-flat directions and an effective R-parity of section 6 are similar to those given in section 4 with minor corrections on the needed singlet VEVs. Section 7 is a conclusion. In appendix A, we list massless spectra according to the sectors. In appendix B, we classify U(1) groups and find out the anomalous $\mathrm{U}(1)_\mathrm{A}$ direction.

2. SSM from $\mathbf{Z}_{12-I}$ compactification

In $E_8\times E_8'$ heterotic orbifold compactification, a model is completely determined with (1) a twist vector ϕ, which is associated with the compactified 3 dimensional complex (or 6 dimensional real) space, (2) a shift vector V which is associated with the 16 dimensional "gauge coordinate" and (3) Wilson line introduced in the compactified space. We employ the $\mathbf{Z}_{12-I}$ orbifold specified with the twist vector $\phi = (\frac{5}{12}\ \frac{4}{12}\ \frac{1}{12})$, and take the following shift vector V and Wilson line a_3:

$$
\begin{aligned}
\phi &= \left(\frac{5}{12}\ \frac{4}{12}\ \frac{1}{12}\right) \\
V &= \left(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ ;\ \frac{5}{12}\ \frac{5}{12}\ \frac{1}{12}\right)\left(\frac{1}{4}\ \frac{3}{4}\ 0\ ;0^5\right)' \\
a_3 &= \left(\frac{2}{3}\ \frac{2}{3}\ \frac{2}{3}\ \frac{-2}{3}\ \frac{-2}{3}\ ;\ \frac{2}{3}\ 0\ \frac{2}{3}\right)\left(0\ \frac{2}{3}\ \frac{2}{3}\ ;0^5\right)'.
\end{aligned}
\tag{2.1}
$$

They satisfy all the conditions required for modular invariance [3, 17] because in our model $V_0^2 - \phi^2 = 1$, $a_3^2 = 4$, $V \cdot a_3 = 1$. They give $V_+^2 - \phi^2 = 7$ and $V_-^2 - \phi^2 = 3$, where $V_{0,+,-} = V + m_f a_3$ with $m_f = 0, +1, -1$.

Low energy field spectrum in a model is determined with (1) massless condition and (2) projection operator. The massless conditions for left and right movers on an orbifold $\mathbf{Z}_N$ are

$$
\begin{aligned}
\text{left movers}: &\quad \frac{(P + kV_f)^2}{2} + \sum_i N_i^L \tilde{\phi}_i - \tilde{c}_k = 0, \\
\text{right movers}: &\quad \frac{(s + k\phi)^2}{2} + \sum_i N_i^R \tilde{\phi}_i - c_k = 0,
\end{aligned}
\tag{2.2}
$$

where $k = 0, 1, 2, \cdots, N-1$, $V_f = (V + m_f a_3)$, and i runs over $\{1, 2, 3, \bar{1}, \bar{2}, \bar{3}\}$. Here $\tilde{\phi}_j \equiv k\phi_j$ mod Z such that $0 < \tilde{\phi}_j \leq 1$, and $\tilde{\phi}_{\bar{j}} \equiv -k\phi_j$ mod Z such that $0 < \tilde{\phi}_{\bar{j}} \leq 1$. If $k\phi_j$ is an integer, $\tilde{\phi}_j = 1$ [9, 10]. N_i^L and N_i^R indicate oscillating numbers for left and right movers. It turns out that $N_i^R = 0$ generically for the massless right mover states in the $\mathbf{Z}_{12-I}$ orbifold compactification. In eqs. (2.2), P and s [$\equiv (s_0, \tilde{s})$] are $E_8 \times E_8'$ and SO(8) weight vectors, respectively. The values of $\tilde{c}_k$, c_k are found in ref. [11].

The multiplicity for a given massless state is calculated by the generalized GSO projection operator [3, 11],

$$
\mathcal{P}_k(f) = \frac{1}{NN_W} \sum_{l=0}^{N-1} \tilde{\chi}(\theta^k, \theta^l) e^{2\pi i l \Theta_f},
\tag{2.3}
$$

where f $(= \{f_0, f_+, f_-\})$ denotes twisted sectors associated with $kV_f = kV$, $k(V + a_3)$, $k(V - a_3)$. N (= 12 in our case) is the order N in the $\mathbf{Z}_N$ orbifold, and N_W is the order of the Wilson line, 3 in our case. The phase Θ_f in eq. (2.3) is given by

$$
\Theta_f = \sum_i (N_i^L - N_i^R)\hat{\phi}_i - \frac{k}{2}(V_f^2 - \phi^2) + (P + kV_f) \cdot V_f - (\tilde{s} + k\phi) \cdot \phi,
\tag{2.4}
$$

where $\hat{\phi}_i = \phi_i\ \mathrm{sgn}(\tilde{\phi}_i)$. Here, $\tilde{\chi}(\theta^k, \theta^l)$ is the degeneracy factor summarized in ref. [11]. Note that $\mathcal{P}_k(f_0) = \mathcal{P}_k(f_+) = \mathcal{P}_k(f_-)$ for $k = 0, 3, 6, 9$.

In addition, the left moving states in the U, T_3, T_6, and T_9 sectors should satisfy [9]

$$
(P + kV) \cdot a_3 = 0 \mod \mathrm{Z}, \quad \text{for} \quad k = 0, 3, 6, 9.
\tag{2.5}
$$

2.1 Massless spectra

With the general formulae eqs. (2.2), (2.3), and (2.5), and our choices eq. (2.1) the massless spectra are calculated.

2.1.1 Chirality and $\mathcal{N}=1$ SUSY

The chirality and the number of supersymmetry (SUSY) $\mathcal{N}$ in four dimensional spacetime (4D) after compactification are determined by the massless right mover states. Massless fermionic states ("R-sector") in the untwisted sector are represented by the four component spinor $s=(s_0;\tilde{s})=(\pm;\pm\pm\pm)$ with even number of plus signs. Throughout this paper, $+$ $(-)$ denotes $\frac{+1}{2}$ $(\frac{-1}{2})$. s_0 determines the chirality of a state. We define a state of $s_0=-\ (+)$ as the left (right) handed state. The corresponding bosonic states ("NS-sector"), which also satisfy the massless condition for the right mover, are obtained just by shifting the left-handed [right-handed] fermionic state by $\tilde{r}_-=(-;-++)$ $[\tilde{r}_+=(+;+--)]$.

The ten dimensional SUSY generators are decomposed into $Q_{(10)}=Q_{(4)}\otimes Q_{(6)}$. Under point group of the orbifold, $Q_{(6)}$ transform as $Q_{(6)}\to\exp(2\pi i s\cdot\phi)Q_{(6)}$. The invariant component corresponds to the unbroken supersymmetry generator in 4D. With $\phi=(\frac{5}{12}\ \frac{4}{12}\ \frac{1}{12})$, the solutions of s satisfying $s\cdot\phi=$integer are only $(-;-++)$ and $(+;+--)$, which give $\mathcal{N}=1$ SUSY because the number of solutions counts the number of unbroken SUSY.

2.1.2 Gauge symmetry and weak mixing angle

The gauge group and gauge quantum numbers are determined by the massless left mover states. The root vectors of $\mathrm{E}_8\times\mathrm{E}_8'$ satisfying $P\cdot V=P\cdot a_3=0$ [3] are only

$$(\underline{1,-1,0};0,0,0^3)(0^8)',\quad (0,0,0;\underline{1,-1};0^8)(0^8)',\quad (0^8)(0^3;\underline{\pm1,\pm1,0,0,0})',\tag{2.6}$$

where the underlined entries allow permutations. Thus the resulting gauge group is

$$\mathrm{SU(3)_c}\times\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}\times\mathrm{U(1)}^4\times[\mathrm{SO(10)}\times\mathrm{U(1)}^3]'.\tag{2.7}$$

Identification of the electroweak hypercharge is essential for the assignment of SM fields, the GUT value of the weak mixing angle $\sin^2\theta_W^0$, the appearance of exotics, and R parity assignments. In this paper, we present two identifications of the electroweak hypercharge: (i) one with exotics and $\sin^2\theta_W^0=\frac{3}{8}$ and (ii) the other without exotics but $\sin^2\theta_W^0=\frac{3}{14}$. The electroweak hypercharge Y is defined as

$$\text{Model E}:\quad Y=\left(\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{-1}{2}\ \frac{-1}{2}\ ;0\ 0\ 0\right)\left(0\ 0\ 0;\ 0\ 0\ 0\ 0\ 0\right)',\tag{2.8}$$

$$\text{Model S}:\quad \tilde{Y}=\left(\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{-1}{2}\ \frac{-1}{2}\ ;0\ 0\ 0\right)\left(0\ 0\ 1;\ 0\ 0\ 0\ 0\ 0\right)',\tag{2.9}$$

where Model E has exotics and $\sin^2\theta_W^0=\frac{3}{8}$ and Model S has only standard $Q_{\rm em}$ charges but $\sin^2\theta_W^0=\frac{3}{14}$. Each assignment has its own merits and shortcomings. The hypercharge Y is orthogonal to every root vector of $\mathrm{SU(3)_c}$, $\mathrm{SU(2)_L}$, and $\mathrm{SO(10)}'$. This operator turns out to give the standard hypercharge assignments to the standard model (SM) chiral fields viz. $Y(Q)=\frac{1}{6}$, etc.

The current algebra in the heterotic string theory fixes the normalization of Y. The $\sin^2\theta_W^0$ estimation is briefed for Model E. Let us consider a properly normalized Z, which is embedded in the string theory as

$$Z = u \times Y = u \times \left[\sqrt{\frac{2}{3}}\, \frac{\vec{q}_3}{\sqrt{2}} - \frac{\vec{q}_2}{\sqrt{2}} \right], \tag{2.10}$$

where u indicates a normalization factor of Y, and $\vec{q}_3$ and $\vec{q}_2$ are orthonormal bases, $\vec{q}_3 = \frac{1}{\sqrt{3}}(1,1,1;0,0;0^3)(0^8)'$ and $\vec{q}_2 = \frac{1}{\sqrt{2}}(0,0,0;1,1;0^3)(0^8)'$. For Z to be embedded in the heterotic string theory, u should be fixed such that $u^2(\frac{2}{3}+1) = 1$ or $u^2 = \frac{3}{5}$ [3, 18]. This hypercharge normalization leads to a gauge coupling normalization $g_1^2 = \frac{5}{3}g_Y^2$, where g_1 is unified at the string scale with the non-Abelian gauge couplings such as $\mathrm{SU(2)_L}$ gauge coupling g_2. Thus, in Model E the weak mixing angle at the string scale is

$$\sin^2\theta_W^0 = \frac{1}{1+(g_2^2/g_Y^2)} = \frac{3}{8}. \tag{2.11}$$

The same kind of calculation gives $\sin^2\theta_W^0 = \frac{3}{14}$ in Model S.

Since $\tilde{Y}$ in Model S is obtained by adding a $\mathrm{U(1)}_6$ generator belonging to E_8', in the bulk of the paper (except section 6) we present quantum numbers of Model E and an effective R-parity. Then, in section 6 we present Model S.

2.1.3 Chiral matter

The matter spectra appear from the untwisted and twisted sectors. All matter fields in this model are tabulated in tables 14–20 in appendix A. Depending on the values of $P \cdot V$, the origins of the fields are denoted by U_1, U_2, U_3 for the untwisted sector fields. We name the twisted sector associated with $kV_f = (V + m_f a_3)$ "$T_k^{m_f}$" with superscripts $0, +, -$ (except for T_3, T_6, T_9). For modular invariance, all these sectors should be considered.

In a $\mathbf{Z}_N$ orbifold compactification, the anti-particle states ($\mathcal{CTP}$ conjugations) of particle states in a $T_k^{m_f}$ sector are, in general, found from the $T_{N-k}^{m_f}$ sector. In the $\mathbf{Z}_{12-I}$ case, the untwisted sector U and T_3, T_6, T_9 sectors provide both left and right chirality states. In particular, the U and T_6 sectors contain particle states and their corresponding anti-particles states. On the other hand, T_1, T_2, T_4, T_7 (T_{11}, T_{10}, T_8, T_5) sectors allow only left (right) chirality states.

As seen in the tables 14–20, this model allows three families of SSM matter fields from the $U_{1,3}$ and T_4^0 sectors. The other fields including the electroweak Higgs are vectorlike under the SM gauge symmetry:

$$3 \times \{Q,\ d^c,\ u^c,\ L,\ e^c,\ \nu^c\} \quad + \quad \text{vectorlike fields (including MSSM Higgs)}. \tag{2.12}$$

The key representations of this SSM are

$$\text{matter} : \begin{cases} Q = (\mathbf{3},\mathbf{2})_{\frac{1}{6}},\ d^c = (\mathbf{3}^*,\mathbf{1})_{\frac{1}{3}},\ u^c = (\mathbf{3}^*,\mathbf{1})_{\frac{-2}{3}}, \\ L = (\mathbf{1},\mathbf{2})_{\frac{-1}{2}},\ e^c = (\mathbf{1},\mathbf{1})_1,\ \nu^c = (\mathbf{1},\mathbf{1})_0, \end{cases} \tag{2.13}$$

$$\text{Higgs} : \begin{cases} H_u = (\mathbf{1},\mathbf{2})_{\frac{1}{2}},\ H_d = (\mathbf{1},\mathbf{2})_{\frac{-1}{2}}, \quad \text{electroweak scale} \\ \mathbf{1}_0, \quad \text{string scale.} \end{cases} \tag{2.14}$$

In this model, there are vectorlike D and $\overline{D}$ (color triplet and antitriplet fields) which carry the familiar d-type quark charge $Q_{\rm em}=\mp\frac{1}{3}$, respectively.

We observe also that there are states with exotic electromagnetic charges (exotics) from the $T_k^{\pm}$ ($k=1,2,4,7$) sectors. All *color exotics* are $\mathrm{SU(3)_c}$ triplets and antitriplets and carry $Q_{\rm em}=0,\pm\frac{1}{3}$. The SU(2) doublet exotics or simply *doublet exotics* carry $Y=\pm\frac{1}{6}$ whose components carry again $Q_{\rm em}=\pm\frac{2}{3},\pm\frac{1}{3}$. The $\mathrm{SU(3)_c}\times\mathrm{SU(2)_L}$ *singlet exotics* carry $Q_{\rm em}=\pm\frac{2}{3},\pm\frac{1}{3}$. *All these exotics form vectorlike representations under the SM gauge symmetry.*[2] The mass scales of these vectorlike representations are near the string scale if the needed neutral singlets develop string scale VEVs. We will comment more on this later.

In table 1, we list particles carrying familiar $Q_{\rm em}$ charges. In addition, we list neutral singlets in table 2. Some of these neutral singlets are required to have string scale VEVs in order to break extra U(1)s and give masses to the exotics.

In the T_3 and T_9 sectors as shown in table 16 of appendix, there are three $\mathbf{10}'$s of $\mathrm{SO}(10)'$. In this model, the hidden sector confining group is $\mathrm{SO}(10)'$. We assume that some of three $\mathbf{10}'$s of $\mathrm{SO}(10)'$ obtain VEVs and break $\mathrm{SO}(10)'$ to a smaller nonabelian group so that its confining scale is at the intermediate scale. The gaugino condensation at this intermediate scale would break the $\mathcal{N}=1$ SUSY.

2.2 Yukawa couplings

To study Yukawa couplings in orbifold compactification, we need to know the H-momentum of a state in a sector. Neglecting the oscillator numbers, the H-momenta of states, $H_{\rm mom,0}$ $[\equiv(\tilde{s}+k\phi+\tilde{r}_-)]$ are

$$\begin{array}{lll}
U_1:\Big(-1,0,0\Big), & U_2:\Big(0,1,0\Big), & U_3:\Big(0,0,1\Big),\\
T_1:\Big(\frac{-7}{12},\frac{4}{12},\frac{1}{12}\Big), & T_2:\Big(\frac{-1}{6},\frac{4}{6},\frac{1}{6}\Big), & T_3:\Big(\frac{-3}{4},0,\frac{1}{4}\Big),\\
T_4:\Big(\frac{-1}{3},\frac{1}{3},\frac{1}{3}\Big), & \Big\{T_5:\Big(\frac{1}{12},\frac{-4}{12},\frac{-7}{12}\Big)\Big\}, & T_6:\Big(\frac{-1}{2},0,\frac{1}{2}\Big),\\
T_7:\Big(\frac{-1}{12},\frac{4}{12},\frac{7}{12}\Big), & T_9:\Big(\frac{-1}{4},0,\frac{3}{4}\Big), &
\end{array} \qquad (2.15)$$

from which T_5 will not be used since the chiral fields there are right-handed while the other fields are represented as left-handed. With oscillators, the H-momentum $[\equiv(R_1,R_2,R_3)]$ are

$$(H_{\rm mom})_j=(H_{\rm mom,0})_j-(N^L)_j+(N^L)_{\bar{j}}\ ,\quad j=1,2,3. \qquad (2.16)$$

The superpotential terms are obtained by examining vertex operators satisfying the orbifold conditions [3]. It can be summarized as the following selection rules:

[2]Since all the SSM matter fields arise from the U and T_4^0 sectors, while all the exotics are only from the twist sectors associated with Wilson line $T_k^{\pm}$ ($k=1,2,4,7$), 3 families of SSM matter fields are relatively easily obtained even with other choices of Wilson line. Indeed, a large class of models with $\frac{1}{4}$ as the first five entries in the shift vector V and with a proper Wilson line can give $\sin^2\theta_W=\frac{3}{8}$ and 3 families of the SSM matter fields. However, it is non-trivial to construct a model such that all exotics form vectorlike representations under the SM gauge symmetry.

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Visible states	SM notation	Γ	Γ'
$(\underline{+\,+\,-};\underline{+-};+\,+\,+)(0^8)'$	$Q(U_1)$	-1	$+1$
$(\underline{+\,-\,-};--;+\,+\,+)(0^8)'$	$d^c(U_3)$	-1	$+1$
$(\underline{+\,-\,-};++;+\,-\,-)(0^8)'$	$u^c(U_3)$	-1	-3
$(-\,-\,-;\underline{+-};+\,-\,-)(0^8)'$	$L(U_1)$	-1	-3
$(+\,+\,+;--;-\,+\,-)(0^8)'$	$e^c(U_3)$	$+5$	$+5$
$(+\,+\,+;++;+\,+\,+)(0^8)'$	$\nu^c(U_3)$	-1	$+1$
$(0\ 0\ 0;\underline{-1\ 0};-1\ 0\ 0)(0^8)'$	$H_u(U_2)$	$+2$	$+2$
$(0\ 0\ 0;\underline{1\ 0};0\ 0\ 1)(0^8)'$	$H_d(U_2)$	-4	-2
$(\underline{+\,+\,-};\underline{+-};\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot Q(T_4^0)$	$+1$	$+1$
$(\underline{+\,-\,-};--;\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot d^c(T_4^0)$	$+1$	$+1$
$(\underline{+\,-\,-};++;\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot u^c(T_4^0)$	-3	-3
$(-\,-\,-;\underline{+-};\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot L(T_4^0)$	-3	-3
$(+\,+\,+;--;\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot e^c(T_4^0)$	$+5$	$+5$
$(+\,+\,+;++;\frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)'$	$2\cdot \nu^c(T_4^0)$	$+1$	$+1$
$(\underline{1,0,0};0\ 0;\frac{-1}{3}\ \frac{-1}{3}\ \frac{1}{3})(0^8)'$	$3\cdot \overline{D}_{1/3}(T_4^0)$	$\boxed{+2}$	$\boxed{+2}$
$(\underline{-1,0,0};0\ 0;\frac{-1}{3}\ \frac{-1}{3}\ \frac{1}{3})(0^8)'$	$2\cdot D_{-1/3}(T_4^0)$	$\boxed{-2}$	$\boxed{-2}$
$(0,0,0;\underline{-1\ 0};\frac{-1}{3}\ \frac{-1}{3}\ \frac{1}{3})(0^8)'$	$2\cdot H_u(T_4^0)$	$+2$	$+2$
$(0,0,0;\underline{1\ 0};\frac{-1}{3}\ \frac{-1}{3}\ \frac{1}{3})(0^8)'$	$3\cdot H_d(T_4^0)$	-2	-2
$(\underline{1,0,0};0\ 0;0^3)(\frac{-1}{2}\ \frac{1}{2}\ 0;0^5)'$	$3\cdot \overline{D}_{1/3}(T_6)$	$\boxed{+2}$	$\boxed{+2}$
$(\underline{-1,0,0};0\ 0;0^3)(\frac{1}{2}\ \frac{-1}{2}\ 0;0^5)'$	$3\cdot D_{-1/3}(T_6)$	$\boxed{-2}$	$\boxed{-2}$
$(0,0,0;\underline{-1\ 0};0^3)(\frac{-1}{2}\ \frac{1}{2}\ 0;0^5)'$	$2\cdot H_u(T_6)$	$+2$	$+2$
$(0,0,0;\underline{1\ 0};0^3)(\frac{1}{2}\ \frac{-1}{2}\ 0;0^5)'$	$2\cdot H_d(T_6)$	-2	-2
$(\underline{\frac{3}{4}\frac{-1}{4}\frac{-1}{4}};\frac{-1}{4}\frac{-1}{4};\frac{1}{4}\frac{1}{4}\frac{1}{4})(\frac{3}{4}\frac{1}{4}\ 0;0^5)'$	$\overline{D}_{1/3}(T_3)$	1	$\boxed{+2}$
$(\underline{\frac{-3}{4}\frac{1}{4}\frac{1}{4}};\frac{1}{4}\frac{1}{4};\frac{-1}{4}\frac{-1}{4}\frac{-1}{4})(\frac{-3}{4}\frac{-1}{4}\ 0;0^5)'$	$2\cdot D_{-1/3}(T_9)$	-1	$\boxed{-2}$
$(\frac{1}{4}\frac{1}{4}\frac{1}{4};\underline{\frac{-3}{4}\frac{1}{4}};\frac{-1}{4}\frac{-1}{4}\frac{-1}{4})(\frac{1}{4}\frac{3}{4}\ 0;0^5)'$	$2\cdot H_u(T_9)$	$+4$	$+3$
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4};\underline{\frac{3}{4}\frac{-1}{4}};\frac{1}{4}\frac{1}{4}\frac{1}{4})(\frac{-1}{4}\frac{-3}{4}\ 0;0^5)'$	$H_d(T_3)$	-4	-3

Table 1: Standard charge left-handed (L) chiral fields. The multiplicity is shown as the coefficients of representations. + and − represent $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. The U(1) charges Γ and Γ' will be presented in eqs. (2.23) and (2.24). Neutral singlets are listed in the following table. $\overline{D}_{1/3}$ and $D_{-1/3}$ in T_4^0 and T_6 have unconventional Γs, not mixing with d and d^c with an exact parity.

(a) Gauge invariance.

(b) H-momentum conservation with $\phi = \left(\frac{5}{12}, \frac{4}{12}, \frac{1}{12}\right)$,

$$\sum_z R_1(z) = -1 \text{ mod } 12, \quad \sum_z R_2(z) = 1 \text{ mod } 3, \quad \sum_z R_3(z) = 1 \text{ mod } 12, \qquad (2.17)$$

where $z(\equiv A, B, C, \dots)$ denotes the index of states participating in a vertex operator.

(c) Space group selection rules:

$$\sum_z k(z) = 0 \text{ mod } 12, \tag{2.18}$$

$$\sum_z [km_f](z) = 0 \text{ mod } 3. \tag{2.19}$$

If some singlets obtain string scale VEVs, however, the condition (b) can be merged into eq. (2.18) in (c). Our strategy to see this is to construct composite singlets (CS) which have H-momenta, (1,0,0), $(-1,0,0)$, (0,1,0), (0,$-$1,0), (0,0,1), (0,0,$-$1), using only singlets developing VEVs of order M_{string}. Then, with any integer set (l,m,n), we can attach an appropriate number of CSs to make the total H-momentum be $(-1,1,1)$. Indeed, it is possible to construct such CSs, with the singlets defined in table 2:

$$\begin{aligned}
[S_1S_8^{(1)}S_{10}][S_4^{(3)}S_7^{(1)}S_{12}][S_4^{(1)}S_7^{(3)}S_{12}] &: \ (1,0,0),\\
[S_1S_8^{(1)}S_{10}][S_4^{(3)}S_7^{(1)}S_{12}][S_1S_8^{(3)}S_{10}] &: \ (-1,0,0),\\
[S_1S_8^{(1)}S_{10}][S_4^{(3)}S_7^{(1)}S_{12}][S_1S_8^{(3)}S_{10}][S_4^{(1)}S_7^{(3)}S_{12}] &: \ (0,1,0),\\
[S_1S_8^{(1)}S_{10}][S_4^{(3)}S_7^{(1)}S_{12}] &: \ (0,-1,0),\\
[S_1S_8^{(1)}S_{10}]^2[S_4^{(3)}S_7^{(1)}S_{12}] &: \ (0,0,1),\\
[S_1S_8^{(1)}S_{10}][S_4^{(3)}S_7^{(1)}S_{12}]^2 &: \ (0,0,-1),
\end{aligned} \tag{2.20}$$

where the CS H-momenta are shown. $S_4^{(1)}$, $S_4^{(3)}$ denote S_4 states with $(N^L)_j = 2_{\bar{1}}, 2_3$, respectively. Similarly, $S_{7,8}^{(1)}$, $S_{7,8}^{(3)}$ are $S_{7,8}$ with $(N^L)_j = 1_{\bar{1}}, 1_3$. For oscillating numbers $(N^L)_j$ of massless states, refer to the tables in appendix A. CS in eq. (2.20) are neutral under all the gauge symmetries in this model, and fulfill the space group selection rules of eqs. (2.18) and (2.19). Hence, multiplication of the above CS to an operator change only the H-momentum vector by integers. Their VEVs are assumed to be of the string scale on a vacuum.

Then, on the vacuum with VEVs for S_1, $S_{4,7,8}^{(1,3)}$, S_{10}, and S_{12}, the H-momentum conservation eq. (2.17) can reduce to

$$\sum_z R_j(z) \Longrightarrow \text{integer}, \quad j = 1,\ 2,\ 3, \tag{2.21}$$

with the understanding that arbitrary number of CS with $\mathcal{O}(M_{\text{string}})$ VEVs can be attached. Thus, if an operator's H-momentum is an integer vector, proper CS can be multiplied such that the resultant H-momentum becomes $(-1,1,1)$ mod $(12,3,12)$. Note that operators multiplied by (higher power of) the above CS are not suppressed, because the VEVs in eq. (2.20) is assumed to be of order M_{string}. Moreover, $(N^L)_j$'s contributions to H-momentum also can be always compensated by proper CS, because they just add integers to $H_{\text{mom},0}$ as seen in eq. (2.16).

H-momentum in T_k sector is generally given by ($H_{\text{mom},0}$ in T_k) = ($H_{\text{mom},0}$ in T_1) $\times$ k + (an integer vector). Accordingly the condition eq. (2.18) is equivalent to eq. (2.21). From now on, we will require only (a) and (c) for Yukawa couplings with the understanding proper CS are multiplied.

Visible states	SM notation	$B-L$	X	Γ	Γ'	Label
$(0\ 0\ 0; 0\ 0; 1\ 0\ \ -1)(0^8)'$	$\mathbf{1_0}(U_2)$	0	0	+2	0	S_0
$(0^5; \frac{-2}{3}\frac{-2}{3}\frac{-1}{3})(\frac{1}{2}\frac{-1}{2}\ 0; 0^5)'$	$\mathbf{1_0}(T_2^0)$	0	0	+2	0	S_1
$(0^5; \frac{-2}{3}\frac{1}{3}\frac{2}{3})(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$\mathbf{1_0}(T_2^0)$	0	0	−2	0	S_2
$(0^5; \frac{1}{3}\frac{-2}{3}\frac{2}{3})(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$\mathbf{1_0}(T_2^0)$	0	0	0	0	S_3
$(0^5; \frac{1}{3}\frac{1}{3}\frac{-1}{3})(\frac{1}{2}\frac{-1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_2^0)$	0	0	0	0	S_4
$(0^5; \frac{1}{3}\frac{1}{3}\frac{-1}{3})(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_2^0)$	0	0	0	0	S_5
$(0^5; \frac{2}{3}\frac{2}{3}\frac{-2}{3})(0^8)'$	$2\cdot\mathbf{1_0}(T_4^0)$	0	0	0	0	S_6
$(0^5; \frac{-1}{3}\frac{-1}{3}\frac{-2}{3})(0^8)'$	$7\cdot\mathbf{1_0}(T_4^0)$	0	0	+2	0	S_7
$(0^5; \frac{-1}{3}\frac{2}{3}\frac{1}{3})(0^8)'$	$6\cdot\mathbf{1_0}(T_4^0)$	0	0	−2	0	S_8
$(0^5; \frac{2}{3}\frac{-1}{3}\frac{1}{3})(0^8)'$	$6\cdot\mathbf{1_0}(T_4^0)$	0	0	0	0	S_9
$(0^5; 1\ 0\ 0)(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_6)$	0	0	0	0	S_{10}
$(0^5; -1\ 0\ 0)(\frac{1}{2}\frac{-1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_6)$	0	0	0	0	S_{11}
$(0^5; 0\ 0\ 1)(\frac{-1}{2}\frac{1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_6)$	0	0	−2	0	S_{12}
$(0^5; 0\ 0\ \ -1)(\frac{1}{2}\frac{-1}{2}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_6)$	0	0	+2	0	S_{13}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{5}{12}\frac{5}{12}\frac{1}{12})(\frac{1}{4}\frac{3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_1^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	0	$\boxed{+1}$	S_{14}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{5}{12}\frac{5}{12}\frac{1}{12})(\frac{-3}{4}\frac{-1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_1^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	$\boxed{-1}$	0	S_{15}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-1}{12}\frac{-1}{12}\frac{-5}{12})(\frac{1}{4}\frac{3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_1^0)$	$-\frac{1}{2}$	$\frac{5}{2}$	$\boxed{+1}$	0	S_{16}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-1}{12}\frac{-1}{12}\frac{-5}{12})(\frac{-3}{4}\frac{-1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_1^0)$	$-\frac{1}{2}$	$\frac{5}{2}$	0	$\boxed{-1}$	S_{17}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{-7}{12}\frac{5}{12}\frac{1}{12})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	$\boxed{-1}$	0	S_{18}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{-7}{12}\frac{5}{12}\frac{1}{12})(\frac{3}{4}\frac{1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	0	$\boxed{+1}$	S_{19}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{5}{12}\frac{-7}{12}\frac{1}{12})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	$\boxed{+1}$	0	S_{20}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{5}{12}\frac{-7}{12}\frac{1}{12})(\frac{3}{4}\frac{1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$\frac{1}{2}$	$-\frac{5}{2}$	+2	$\boxed{+1}$	S_{21}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-1}{12}\frac{-1}{12}\frac{7}{12})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$-\frac{1}{2}$	$\frac{5}{2}$	−2	$\boxed{-1}$	S_{22}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-1}{12}\frac{-1}{12}\frac{7}{12})(\frac{3}{4}\frac{1}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_7^0)$	$-\frac{1}{2}$	$\frac{5}{2}$	$\boxed{-1}$	0	S_{23}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{-3}{4}\frac{1}{4}\frac{1}{4})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_3)$	$-\frac{1}{2}$	$\frac{5}{2}$	−2	$\boxed{-1}$	S_{24}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{3}{4}\frac{-1}{4}\frac{-1}{4})(\frac{1}{4}\frac{3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_9)$	$\frac{1}{2}$	$-\frac{5}{2}$	+2	$\boxed{+1}$	S_{25}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{1}{4}\frac{1}{4}\frac{-3}{4})(\frac{-1}{4}\frac{-3}{4}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_3)$	$-\frac{1}{2}$	$\frac{5}{2}$	0	$\boxed{-1}$	S_{26}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{-1}{4}\frac{-1}{4}\frac{3}{4})(\frac{1}{4}\frac{3}{4}\ 0; 0^5)'$	$\mathbf{1_0}(T_9)$	$\frac{1}{2}$	$-\frac{5}{2}$	0	$\boxed{+1}$	S_{27}
$(\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}; \frac{-1}{4}\frac{-1}{4}\frac{-1}{4})(\frac{3}{4}\frac{1}{4}\ 0; 0^5)'$	$2\cdot\mathbf{1_0}(T_3)$	$\frac{1}{2}$	$-\frac{5}{2}$	+2	$\boxed{+1}$	S_{28}
$(\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}\frac{-1}{4}; \frac{1}{4}\frac{1}{4}\frac{1}{4})(\frac{-3}{4}\frac{-1}{4}\ 0; 0^5)'$	$3\cdot\mathbf{1_0}(T_9)$	$-\frac{1}{2}$	$\frac{5}{2}$	−2	$\boxed{-1}$	S_{29}

Table 2: Left-handed electromagnetically neutral SO(10)′ singlets. There is only one untwisted sector singlet S_0. To have a definition of parity, S_{15}, S_{16}, S_{18}, S_{20}, and S_{23} should not develop VEVs.

2.2.1 Phenomenologically desirable vacuum

The phenomenologically desirable SSM vacuum is chosen by assigning nonzero VEVs to *some* SM singlet fields such that

- unwanted exotics achieve heavy enough masses,
- U(1) gauge symmetries that are not observed at low energies are broken, and

- R-parity violating couplings inducing too rapid proton decay are sufficiently suppressed.

All the neutral singlets appearing in this model are listed in table 2.

To attain the aims mentioned above, let us *choose a vacuum*, as one possibility, on which the following neutral singlets get vanishing or non-vanishing VEVs:

$$\begin{aligned}&\langle S_0\rangle \neq 0,\ \langle S_1\rangle \neq 0,\ \cdots,\ \langle S_{13}\rangle \neq 0,\ \langle S_{15}\rangle \neq 0,\ \langle S_{23}\rangle \neq 0,\ \langle S_{29}\rangle \neq 0 \\ &\langle S_{14}\rangle = \langle S_{16}\rangle = \langle S_{17}\rangle = \cdots = \langle S_{22}\rangle = \langle S_{24}\rangle = \langle S_{25}\rangle = \cdots = \langle S_{28}\rangle = 0.\end{aligned} \tag{2.22}$$

In section 3, we will show that the non-vanishing VEVs in eqs. (2.22) are enough to give heavy masses to all the exotics present in this model.

The VEVs of the singlets in eq. (2.22) break U(1) symmetries in eq. (2.7) except $\mathrm{U}(1)_\mathrm{Y}$ and $\mathrm{U}(1)_6$, since all the neutral singlets don't carry the charges of $\mathrm{U}(1)_\mathrm{Y}$ and $U(1)_6$. The $\mathrm{U}(1)_6$ generator is defined as $Q_6 = (0^8)(0,0,2;0^5)'$. In fact, all Q_6 nonzero charges are carried only by the exotics as shown in tables 14–20. All the observable matter fields are neutral under $\mathrm{U}(1)_6$. Thus, in addition to photon there exists another strictly massless $U(1)_6$ gauge boson which is named as *exotic photon* (*exphoton* for abbreviation). Since it couples only to superheavy exotic matter, the presence of the "exphoton" is phenomenologically acceptable.

In tables 1 and 2, we displayed the $\mathrm{U}(1)_\Gamma$ and $\mathrm{U}(1)_{\Gamma'}$ quantum numbers. The $\mathrm{U}(1)_\Gamma$ and $\mathrm{U}(1)_{\Gamma'}$ are linear combinations of U(1)s observed in this model. Their generators are defined as

$$\Gamma = X - (Q_2 + Q_3) + \frac{1}{4}(Q_4 + Q_5) + 6(B - L), \tag{2.23}$$

$$\Gamma' = X + \frac{1}{4}(Q_4 + Q_5) + 6(B - L), \tag{2.24}$$

where

$$\begin{aligned} Q_2 &= (0^5;0,2,0)(0^8)', \quad Q_3 = (0^5;0,0,2)(0^8)' \\ Q_4 &= (0^8)(2,0,0;0^5)', \quad Q_5 = (0^8)(0,2,0;0^5)' \\ X &= (-2,-2,-2,-2,-2;0^3)(0^8)' \\ B - L &= \left(\frac{2}{3},\frac{2}{3},\frac{2}{3},0^5\right)(0^8)' \end{aligned} \tag{2.25}$$

Q_4 and Q_5 depend only on the hidden E_8'. The $\mathrm{U}(1)_\Gamma$ symmetry will be used in section 5 for a discussion on R-parity. We put boxes for $\Gamma^{(\prime)} = \pm 1$ singlet fields. A desirable vacuum toward an exact R-parity might be the one with vanishing VEVs for all these boxed singlets. If the R-parity is not exact, it should be an approximate symmetry valid at low energy processes. These conditions should, of course, be consistent with other phenomenological requirements such as large (small) enough exotic mass terms (μ term). In table 1, we also boxed some D and $\overline{D}$ fields which have different type $\mathrm{U}(1)_\Gamma$ quantum numbers from those of d and d^c quarks. Namely, if the parity defined from $\mathrm{U}(1)_\Gamma$ is exact, these D and $\overline{D}$ do not mix with light quarks d and d^c.

Note that the neutral singlets developing VEVs in eq. (2.22) carry only zero or negative Γ' charges: $\Gamma' = 0$ or -1. In fact, $\langle S_{29} \rangle$ break the parity, however, in section 5 we will also show that the light fields can still have a useful approximate R-parity.

2.2.2 The third family in the untwisted sector

Sixteen chiral fields in eq. (2.13) form a family. One family appears in the untwisted sectors, U_1 and U_3. $\mathrm{SU}(2)_{\mathrm{L}}$ doublets are in U_1 and $\mathrm{SU}(2)_{\mathrm{L}}$ singlets are in U_3. The remaining two families arise from T_4^0. Since the third family quarks are unique in being heavy, we assign the third family to the untwisted sector fields. Indeed, there can exist cubic couplings for the untwisted sector family by the coupling $U_1U_2U_3$ allowed by the original selection rules (a), (b), and (c). For this to be a viable interpretation, H_u and H_d in U_2 must survive down to the electroweak scale.

2.2.3 Light families and mixing angles

With the VEVs of eq. (2.22), the (reduced) selection rules allow also the mass terms of the first two families of the SSM chiral matter. For example, Q and d^c in the T_4^0 sector can couple together with S_7 or S_1S_5, if the oscillating number carried by S_7 or S_1S_5 is compensated by a proper CS. The cross terms, $Q(U_1)$-$d^c(T_4^0)$ and $Q(T_4^0)$-$d^c(U_3)$ are also possible through S_7^2·CS (or $[S_1S_5]^2$·CS). Thus the $d^c - d$ mass matrix, $M^{(d)}$ takes the form

$$\begin{array}{c} \\ d^c(T_4^0) \\ d^c(T_4^0) \\ d^c(U_3) \end{array} \begin{array}{c} \begin{array}{ccc} Q(T_4^0) & Q(T_4^0) & Q(U_1) \end{array} \\ \left(\begin{array}{ccc} a & b & x^{(d)} \\ b & a & x^{(d)} \\ x^{(d)} & x^{(d)} & z \end{array} \right) \end{array} \langle H_d(U_2) \rangle,$$

where $z = S_7^3$ (or $[S_1S_5]^3$) and $x^{(d)} = S_7^2$ (or $[S_1S_5]^2$). Here we set $\langle \mathrm{CS} \rangle = 1$. The down-type quark mass matrix is symmetric. For flavor democratic T_4^0 couplings, we have a common entry a instead of a, b in the 2×2 sub-matrix. But a flavor democratic form is one specific representation of the $\mathbf{S}_2$ permutation symmetry. For a general $\mathbf{S}_2$ representation for T_4^0 sector fields, the upper left 2×2 sub-matrix is of the form given above. So, in general its determinant is nonzero. To have nonzero mixing angles, the up-type quark mass matrix, $M^{(u)}$, should not align to the down-type quark mass matrix, $M^{(d)}$. The up-type $u^c - u$ quark mass matrix is

$$\begin{array}{c} \\ u^c(T_4^0) \\ u^c(T_4^0) \\ u^c(U_3) \end{array} \begin{array}{c} \begin{array}{ccc} Q(T_4^0) & Q(T_4^0) & Q(U_1) \end{array} \\ \left(\begin{array}{ccc} a' & b' & x^{(u)} \\ b' & a' & x^{(u)} \\ y^{(u)} & y^{(u)} & 1 \end{array} \right) \end{array} \langle H_u(U_2) \rangle,$$

where

$$\{a', b'\} = \{S_9,\ S_3S_4\},\quad x^{(u)} = \{S_7S_9,\ S_1S_5S_9,\ S_7S_3S_4\},\quad y^{(u)} = \{S_8S_9,\ S_2S_4S_9,\ S_8S_3S_4\}.$$

a' and b' which are linear combinations of S_9 and S_3S_4 can be different in principle. In $M^{(u)}$, proper CS multiplications are assumed. Unlike $M^{(d)}$, $M^{(u)}$ is not symmetric.

Similarly, the charged lepton mass matrix $M^{(e)}$ is

$$\begin{array}{c} \\ e^c(T_4^0) \\ e^c(T_4^0) \\ e^c(U_3) \end{array} \begin{array}{c} \begin{array}{ccc} L(T_4^0) & L(T_4^0) & L(U_1) \end{array} \\ \left(\begin{array}{ccc} a'' & b'' & x^{(e)} \\ b'' & a'' & x^{(e)} \\ y^{(e)} & y^{(e)} & 1 \end{array} \right) \end{array} \langle H_d(U_2) \rangle,$$

where

$$\{a'', b''\} = \{S_7,\ S_1S_5\},\ \ x^{(e)} = \{S_7S_8,\ S_1S_5S_8,\ S_7S_2S_4\},\ \ y^{(e)} = \{S_7S_9,\ S_1S_5S_9,\ S_7S_3S_4\}.$$

Neutrinos obtain mass. With the following Dirac and Majorana mass terms, the seesaw type light neutrino masses are possible:

$$\text{Dirac}: \begin{array}{c} \\ \nu^c(T_4^0) \\ \nu^c(T_4^0) \\ \nu^c(U_3) \end{array} \begin{array}{c} \begin{array}{ccc} L(T_4^0) & L(T_4^0) & L(U_1) \end{array} \\ \left(\begin{array}{ccc} c & c & x^{(\nu)} \\ c & c & x^{(\nu)} \\ y^{(\nu)} & y^{(\nu)} & 1 \end{array} \right) \end{array} \langle H_u(U_2) \rangle,$$

$$\text{Majorana}: \begin{array}{c} \\ \nu^c(T_4^0) \\ \nu^c(T_4^0) \\ \nu^c(U_3) \end{array} \begin{array}{c} \begin{array}{ccc} \nu^c(T_4^0) & \nu^c(T_4^0) & \nu^c(U_3) \end{array} \\ \left(\begin{array}{ccc} M_2 & M_2 & M_1 \\ M_2 & M_2 & M_1 \\ M_1 & M_1 & M_0 \end{array} \right) \end{array}$$

where

$$c = \{S_9,\ S_3S_4\}\ \ x^{(\nu)} = \{S_8S_9,\ S_2S_4S_9,\ S_8S_3S_4\},\ \ y^{(\nu)} = \{S_7S_9, S_1S_5S_9,\ S_7S_3S_4\},$$

and

$$M_0 = [S_{23}S_{29}]^2[S_7]^4,\ \ M_1 = [S_{23}S_{29}]^2[S_7]^3,\ \ M_2 = [S_{23}S_{29}]^2[S_7]^2.$$

Therefore, the vacuum (2.22) can give successful quark and lepton mass matrices.

2.2.4 Higgs doublets and μ term

Vectorlike electroweak doublet fields, $H_u(Y = \frac{1}{2})$ and $H_d(Y = -\frac{1}{2})$, appear in U_2, T_4^0, T_6, T_3, and T_9. The selection rules (b) and (c) in section (2.2) allow interactions of $U_2U_2\times$CS and $U_2U_2T_6T_6\times$CS. Among these interactions, $[H_u(U_2)H_d(U_2)] \times (S_0 \cdot \text{CS} + S_{10}S_{13} \cdot \text{CS})$ are present. We regard $\{H_u(U_2), H_d(U_2)\}$ as the MSSM Higgs fields. TeV scale VEV of $(S_0 \cdot \text{CS} + S_{10}S_{13} \cdot \text{CS})$ gives the MSSM "μ" term. We will discuss it again later.

The selection rules permit $T_6T_6\times$CS couplings. So, $H_u(T_6)H_d(T_6)\times$CS couplings are present. Hence two pairs of H_u and H_d from T_6 obtain heavy mass by string scale VEV of CS. The selection rules admit also $T_4^0T_4^0T_4^0$ couplings. So there exist $H_u(T_4^0)H_d(T_4^0)S_6(T_4^0)$ couplings, from which two pairs of H_u and H_d in T_4^0 also become heavy by string scale VEVs of S_6.[3] There remains one $H_d(T_4^0)$ at this level.

[3]We ignore a possible permutation symmetry at this level of study.

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Pairs	Masses (×proper CS)
$\{H_u(U_2),\ H_d(U_2)\}$	$S_0,\ S_{10}S_{13}$
$\{H_u(U_2),\ H_d(T_6)\}$	S_{10}
$\{H_u(U_2),\ H_d(T_4^0)\}$	$S_{10}S_4$
$\{H_u(U_2),\ H_d(T_3)\}$	0
$\{H_u(T_6),\ H_d(U_2)\}$	S_{13}
$\{H_u(T_6),\ H_d(T_6)\}$	1
$\{H_u(T_6),\ H_d(T_4^0)\}$	S_4
$\{H_u(T_6),\ H_d(T_3)\}$	0
$\{H_u(T_4^0),\ H_d(U_2)\}$	$S_{13}S_5$
$\{H_u(T_4^0),\ H_d(T_6)\}$	S_5
$\{H_u(T_4^0),\ H_d(T_4^0)\}$	S_6
$\{H_u(T_4^0),\ H_d(T_3)\}$	0
$\{H_u(T_9),\ H_d(U_2)\}$	$S_{13}S_{29}$
$\{H_u(T_9),\ H_d(T_6)\}$	S_{29}
$\{H_u(T_9),\ H_d(T_4^0)\}$	S_4S_{29}
$\{H_u(T_9),\ H_d(T_3)\}$	1

Table 3: Mass terms for H_u and H_d. CS are products of singlet fields given in eq. (2.19). Proper CS are assumed to be multiplied such that the H-momentum becomes $(-1,1,1)$ mod $(12,3,12)$. We set $\langle \text{CS}\rangle = 1$. For μ solution we assume that a modulus is involved in S_0 or $S_{10}S_{13}$.

$T_3T_9\times$CS couplings are also allowed. Thus, there exist couplings of $H_u(T_9)H_d(T_3)\times$CS, and by a VEV of CS one pair of $\{H_u(T_9), H_d(T_3)\}$ is made heavy. Thus, there remains one $H_u(T_9)$ also at this level.

The remaining H_d in T_4^0 and H_u in T_9 can also be made heavy via the coupling $[H_u(T_9)H_d(T_4^0)]\times\langle S_4S_{29}\rangle$. This coupling is one of $T_9T_4^0T_2^0T_9$ interactions, which satisfies the selection rules. To study the masses in more detail, we list the full H_uH_d couplings in table 3.

Now we can represent a schematic form of the 7×7 H_uH_d mass matrix as

$$\begin{array}{c} \\ H^0 \\ H^6 \\ H^6 \\ H^4 \\ H^4 \\ H^9 \\ H^9 \end{array} \begin{array}{c} \quad H_0\ \ H_6\ \ H_6\ \ H_4\ \ H_4\ \ H_4\ \ H_3 \\ \begin{pmatrix} \triangle & \star & \star & \star' & \star' & \star' & 0 \\ * & \times & \times & > & > & > & 0 \\ * & \times & \times & > & > & > & 0 \\ *' & < & < & \vee & \vee & \vee & 0 \\ *' & < & < & \vee & \vee & \vee & 0 \\ *'' & \Diamond & \Diamond & \Diamond' & \Diamond' & \Diamond' & \times \\ *'' & \Diamond & \Diamond & \Diamond' & \Diamond' & \Diamond' & \times \end{pmatrix} \end{array} . \tag{2.26}$$

Here H^0, H^6, H^4, and H^9 indicate $H_u(U_2)$, $H_u(T_6)$, $H_u(T_4)$, and $H_u(T_9)$, respectively. Similarly, $H_0 \equiv H_d(U_2)$, $H_4 \equiv H_d(T_4^0)$, and $H_3 \equiv H_d(T_3)$. $\triangle$ denotes non-vanishing VEVs by S_0 and $S_{10}S_{13}$, $\triangle \equiv S_0 + S_{10}S_{13}$. As mentioned earlier, we tacitly assume proper VEVs of CS, which are of string scale, are multiplied to fulfill the selection rule (b) discussed in section 2.2. $\times$s stand for non-vanishing VEVs by CS. $\star$ and $\star'$ are VEVs of S_{10} and $S_{10}S_4$, and $*$, $*'$, $*''$ are those of S_{13}, $S_{13}S_5$, $S_{13}S_{29}$. $>$, $<$, and $\vee$ correspond to VEVs of S_4, S_5, and S_6, respectively. $\Diamond$ and $\Diamond'$ are VEVs of S_{29} and S_4S_{29}. Since any neutral singlets with non-vanishing VEVs do not carry positive Γ' charges, zero entries in the above matrix eq. (2.26), which are associated with $H_d(T_3)$, should be exactly zeros.

We suppose relatively small VEVs for S_{10} and S_{13} compared to the other VEVs of neutral singlets:

$$S_{10},\ S_{13}\ \lesssim\ \mathcal{O}(M_{\rm string}). \tag{2.27}$$

Then the mixing angle between $\{H_u(U_2), H_d(U_2)\}$ and the other H_u-H_d pairs is suppressed, and the effective "μ" coefficient of $H_u(U_2)H_d(U_2)$ is estimated as

$$\mu \sim S_0 + \mathcal{O}(S_{10}S_{13}/M_{\rm string}). \tag{2.28}$$

If one VEV among S_0, S_{10}, and S_{13} is left undetermined at the string scale, μ is also undetermined in the SUSY limit. With soft SUSY breaking terms, however, μ (and Higgs VEVs) could be fixed around TeV scale. In the limit $\mu \to 0$, an accidental Peccei-Quinn symmetry revives. We do not discuss it in this paper.

2.2.5 Vectorlike $D^{-1/3}$ and $\overline{D}^{1/3}$

The $Q_{\rm em}= \mp\frac{1}{3}$ colored fields $D^{-1/3}$ and $\overline{D}^{1/3}$ appear only in twisted sectors T_6, T_4^0 T_3, and T_9. Three pairs of $\{D(T_6)$ and $\overline{D}(T_6)\}$ can be removed from low energy field spectra via $D(T_6)\overline{D}(T_6)\times$CS.

The coupling $D(T_4^0)\overline{D}(T_4^0)S_6(T_4^0)$ remove two pairs of D and $\overline{D}$ in T_4^0, leaving one $\overline{D}$ in T_4^0. The coupling of the form $D(T_9)\overline{D}(T_3)\times$CS is present, and so one pair of D and $\overline{D}$ is removed at this level, leaving one D in T_9.

The remaining $\overline{D}(T_4^0)$ and $D(T_9)$ can be heavy via the two couplings $[D(T_9)\overline{D}(T6)] \times \langle S_4S_{23}\cdot{\rm CS}\rangle$ and $[D(T6)\overline{D}(T_4^0)] \times \langle S_5\cdot{\rm CS}\rangle$. Note that here $D(T_6)$ and $\overline{D}(T_6)$ are already coupled to each other to have the mass term with a VEV of CS. Therefore it is obvious that all $\{D, \overline{D}\}$ obtain masses. We list all D-$\overline{D}$ couplings in table 4.

The 8×8 D-$\overline{D}$ mass matrix is of the form

$$\begin{array}{c} \\ D_6 \\ D_6 \\ D_6 \\ D_4 \\ D_4 \\ D_9 \\ D_9 \end{array}
\begin{array}{c} \begin{array}{ccccccc} \overline{D}_6 & \overline{D}_6 & \overline{D}_6 & \overline{D}_4 & \overline{D}_4 & \overline{D}_4 & \overline{D}_3 \end{array} \\
\left(\begin{array}{ccccccc}
\times & \times & \times & < & < & < & 0 \\
\times & \times & \times & < & < & < & 0 \\
\times & \times & \times & < & < & < & 0 \\
> & > & > & \vee & \vee & \vee & \boxtimes \\
> & > & > & \vee & \vee & \vee & \boxtimes \\
\square & \square & \square & \square' & \square' & \square' & \times \\
\square & \square & \square & \square' & \square' & \square' & \times
\end{array}\right) \end{array}, \tag{2.29}$$

where $D_6, \overline{D}_4$, etc. mean $D(T_6), \overline{D}(T_4)$, etc. $\times$, $<$, $>$, and $\vee$ entries stand again for VEVs of CS, S_5, S_4, and S_6, respectively. $\square$, $\square'$, and $\boxtimes$ denote VEVs of $S_{23}S_4$, $S_{23}S_6$, and S_7S_{15}.

Pairs	Masses (×proper CS)
$\{D(T_6),\ \overline{D}(T_6)\}$	1
$\{D(T_6),\ \overline{D}(T_4)\}$	S_5
$\{D(T_6),\ \overline{D}(T_3)\}$	0
$\{D(T_4),\ \overline{D}(T_6)\}$	S_4
$\{D(T_4),\ \overline{D}(T_4)\}$	S_6
$\{D(T_4),\ \overline{D}(T_3)\}$	$S_7 \boxed{S_{15}}$
$\{D(T_9),\ \overline{D}(T_6)\}$	$\boxed{S_{23}} S_4$
$\{D(T_9),\ \overline{D}(T_4)\}$	$\boxed{S_{23}} S_6$
$\{D(T_9),\ \overline{D}(T_3)\}$	1

Table 4: Mass terms for D and $\overline{D}$. CS are products of singlet fields given in eq. (2.19). Proper CS are assumed to be multiplied such that the H-momentum becomes $(-1, 1, 1)$ mod $(12, 3, 12)$. We set $\langle \mathrm{CS} \rangle = 1$.

Through the mass terms in eq. (2.29), all Ds and $\overline{D}$s are paired to be superheavy. Mixing terms between d^cs in U_3, T_4^0 and Ds in T_6, T_4^0, T_9 can not arise in any manner. It is because the negative Γ' charges carried by such mixing terms cannot be compensated by neutral singlets with non-zero VEVs.

This shows that the odd Γ and $Q_{\rm em}= -\frac{1}{3}$ quarks of table 1 can mix among themselves, but not with $D(T_6)$, $D(T_4^0)$, $\overline{D}(T_6)$ and $\overline{D}(T_4^0)$, in the limit $S_{23} \to 0$. So, the down-type quarks have additional contribution to the mass matrix by mixing with $D(T_9)$ and $\overline{D}(T_3)$, and non-vanishing quark mixing is achieved in general.

Even if $S_{15} = 0$ (so $\boxtimes = 0$), all D and $\overline{D}$ still obtain masses because the determinant of eq. (2.29) is nonzero. If $S_{23} = 0$ (so $\Box = \Box' = 0$), however, the above type mass mixing does not give a mass to one pair of D-$\overline{D}$. Hence it seems necessary to have at least one Γ odd singlet obtain a VEV. Let us choose the VEV $\langle S_{23} \rangle$ as the parameter contributing to P violating terms among the low energy fields.

3. Vectorlike exotics

Among the phenomenological conditions, the exotics mass condition must be satisfied at any cost. In this model, exotic fields appears in the $T_1^{\pm}$, $T_2^{\pm}$, $T_4^{\pm}$, and $T_7^{\pm}$ (or $T_5^{\pm}$) sectors. The color triplet exotics carry the electromagnetic charges of $0, \pm\frac{1}{3}$. The doublet and singlet exotics carry also fractional electromagnetic charges: $Q_{\rm em}= \pm\frac{2}{3}, \mp\frac{1}{3}$. Color exotics could form color singlet states with fractional electromagnetic charges. Searches for fractionally charged particles have not given any positive evidence, and hence all exotics on the vacuum we choose should be heavy enough. Let us proceed to discuss how the vectorlike exotic states achieve masses.

3.1 Color exotics

In table (5), we list the color exotics found in our model. They are singlets under $\mathrm{SU}(2)_{\rm L}$.

Color exotics	$SU(3)_c$(Sector)	$(\alpha \text{ or } \overline{\alpha})^{Q_{em}}$
$\left(\underline{\tfrac{-7}{12},\tfrac{5}{12},\tfrac{5}{12}};\tfrac{1}{12},\tfrac{1}{12};\tfrac{-5}{12},\tfrac{-1}{12},\tfrac{3}{12}\right)\left(\tfrac{3}{12},\tfrac{5}{12},\tfrac{-4}{12};0^5\right)'$	$\mathbf{3}(T_1^+)$	α_1^0
$\left(\underline{\tfrac{5}{6},\tfrac{-1}{6},\tfrac{-1}{6}};\tfrac{1}{6},\tfrac{1}{6};\tfrac{1}{6},\tfrac{-1}{6},\tfrac{-1}{2}\right)\left(\tfrac{1}{2},\tfrac{-1}{6},\tfrac{1}{3};0^5\right)'$	$\mathbf{3}^*(T_2^+)$	$\overline{\alpha}_2^0$
$\left(\underline{\tfrac{-5}{6},\tfrac{1}{6},\tfrac{1}{6}};\tfrac{-1}{6},\tfrac{-1}{6};\tfrac{-1}{2},\tfrac{-1}{6},\tfrac{-1}{6}\right)\left(\tfrac{-1}{2},\tfrac{1}{6},\tfrac{-1}{3};0^5\right)'$	$\mathbf{3}(T_2^-)$	α_3^0
$\left(\underline{\tfrac{-5}{6},\tfrac{1}{6},\tfrac{1}{6}};\tfrac{-1}{6},\tfrac{-1}{6};\tfrac{-1}{6},\tfrac{1}{6},\tfrac{-1}{2}\right)\left(0,\tfrac{-1}{3},\tfrac{-1}{3};0^5\right)'$	$\mathbf{3}(T_4^+)$	$2\cdot\alpha_4^0$
$\left(\underline{\tfrac{5}{6},\tfrac{-1}{6},\tfrac{-1}{6}};\tfrac{1}{6},\tfrac{1}{6};\tfrac{-1}{2},\tfrac{1}{6},\tfrac{1}{6}\right)\left(0,\tfrac{1}{3},\tfrac{1}{3};0^5\right)'$	$\mathbf{3}^*(T_4^-)$	$3\cdot\overline{\alpha}_5^0$
$\left(\underline{\tfrac{2}{3},\tfrac{-1}{3},\tfrac{-1}{3}};\tfrac{1}{3},\tfrac{1}{3};\tfrac{1}{3},\tfrac{-1}{3},0\right)\left(0,\tfrac{-1}{3},\tfrac{-1}{3};0^5\right)'$	$\mathbf{3}^*(T_4^+)$	$2\cdot\overline{\alpha}_6^{-1/3}$
$\left(\underline{\tfrac{-2}{3},\tfrac{1}{3},\tfrac{1}{3}};\tfrac{-1}{3},\tfrac{-1}{3};0,\tfrac{-1}{3},\tfrac{-1}{3}\right)\left(0,\tfrac{1}{3},\tfrac{1}{3};0^5\right)'$	$\mathbf{3}(T_4^-)$	$2\cdot\alpha_7^{1/3}$

Table 5: Color exotics of $Q_{em}= 0, \pm\frac{1}{3}$. Color $\mathbf{3}$ and $\mathbf{3}^*$ with $Q_{em}= \pm\frac{1}{3}$ are exotics.

Pairs	Masses (×proper CS)
$1\times\{\alpha_3^0(T_2^-),\ \overline{\alpha}_2^0(T_2^+)\}$	S_4S_{12}
$2\times\{\alpha_4^0(T_4^+),\ \overline{\alpha}_5^0(T_4^-)\}$	$S_9,\ S_3S_4$
$1\times\{\alpha_1^0(T_1^+),\ \overline{\alpha}_5^0(T_4^-)\}$	$S_9S_{13}S_{29}$
$2\times\{\alpha_7^{1/3}(T_4^-),\ \overline{\alpha}_6^{-1/3}(T_4^+)\}$	$S_8,\ S_2S_4$

Table 6: Mass terms for color exotics. CS are products of singlet fields given in eq. (2.19). Proper CS are assumed to be multiplied such that the H-momentum becomes $(-1, 1, 1)$ mod $(12, 3, 12)$. We set $\langle\text{CS}\rangle = 1$.

As seen in the table, the color exotics are vectorlike under the SM gauge symmetry. They all can achieve masses when the neutral singlets in eq. (2.22) get VEVs. To prove this, we don't have to study the full mass matrix for the vectorlike exotics. Instead, we will suggest just some couplings enough to show that they are heavy. In table 6, we present the minimal number of couplings yielding their masses. Since all the vectorlike exotics in table 5 can pair up with neutral singlets, they can be removed from low energy field spectra.

3.2 Doublet exotics

In this model there are $SU(2)_L$ doublet fields carrying exotic electromagnetic charges. They are $SU(3)_c$ singlets but possess the charges of $Y = \pm\frac{1}{6}$ (or $Q_{em} = \pm\frac{2}{3},\ \mp\frac{1}{3}$). In table 7, all doublet exotics are collected. All the vectorlike doublet exotics in table 7 could achieve masses via couplings with neutral singlets developing VEVs. The minimal number of couplings for them to be heavy are displayed in table 8. Hence, all the doublet exotics can obtain masses.

3.3 Singlet exotics

There are 38 kinds (in terms of gauge quantum numbers) of singlet exotics, as collected in tables 9. In these tables, $\xi, \overline{\xi}$ are $Q_{em}= \pm\frac{2}{3}$ singlets and $\eta, \overline{\eta}$ are $Q_{em}= \mp\frac{1}{3}$ singlets.

Doublet exotics	$[\mathrm{SU}(2)_{\mathrm{L}}]^{Y}$ (Sector)	Label
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\underline{\frac{7}{12},\frac{-5}{12}};\frac{1}{12},\frac{5}{12},\frac{-3}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	$\mathbf{2}^{-1/6}(T_1^+)$	$\overline{\delta}_1$
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\underline{\frac{-7}{12},\frac{5}{12}};\frac{3}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{-9}{12},\frac{1}{12},\frac{4}{12};0^5\right)'$	$\mathbf{2}^{1/6}(T_1^-)$	δ_2
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\underline{\frac{-7}{12},\frac{5}{12}};\frac{3}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{3}{12},\frac{1}{12},\frac{-8}{12};0^5\right)'$	$\mathbf{2}^{1/6}(T_1^-)$	δ_3
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\underline{\frac{5}{6},\frac{-1}{6}};\frac{-1}{6},\frac{1}{6},\frac{-1}{2}\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	$3\cdot\mathbf{2}^{-1/6}(T_4^+)$	$\overline{\delta}_4$
$\left(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3};\underline{\frac{-2}{3},\frac{1}{3}};\frac{1}{3},\frac{-1}{3},0\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	$2\cdot\mathbf{2}^{-1/6}(T_4^+)$	$\overline{\delta}_5$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\underline{\frac{-5}{6},\frac{1}{6}};\frac{-1}{2},\frac{1}{6},\frac{1}{6}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	$2\cdot\mathbf{2}^{1/6}(T_4^-)$	δ_6
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\underline{\frac{2}{3},\frac{-1}{3}};0,\frac{-1}{3},\frac{-1}{3}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	$2\cdot\mathbf{2}^{1/6}(T_4^-)$	δ_7
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\underline{\frac{7}{12},\frac{-5}{12}};\frac{1}{12},\frac{-7}{12},\frac{-3}{12}\right)\left(\frac{-3}{12},\frac{-1}{12},\frac{8}{12};0^5\right)'$	$\mathbf{2}^{-1/6}(T_7^+)$	$\overline{\delta}_8$
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\underline{\frac{-7}{12},\frac{5}{12}};\frac{-9}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{-3}{12},\frac{-5}{12},\frac{4}{12};0^5\right)'$	$\mathbf{2}^{1/6}(T_7^-)$	δ_9

Table 7: SU(2) doublet exotics with $Q_{\mathrm{em}}=\pm\frac{2}{3},\mp\frac{1}{3}$.

Pairs	Masses ($\times$proper CS)
$1\times\{\overline{\delta}_4(T_4^+),\ \delta_2(T_1^-)\}$	$\boxed{S_{23}}$
$2\times\{\overline{\delta}_5(T_4^+),\ \delta_7(T_4^-)\}$	$S_8,\ S_2S_4$
$1\times\{\overline{\delta}_1(T_1^+),\ \delta_9(T_7^-)\}$	$S_9,\ S_3S_4$
$1\times\{\overline{\delta}_8(T_7^+),\ \delta_3(T_1^-)\}$	$S_8,\ S_2S_4$
$2\times\{\overline{\delta}_4(T_4^+),\ \delta_6(T_4^-)\}$	$S_9,\ S_3S_4$

Table 8: Mass terms for doublet exotics. CS are products of singlet fields given in eq. (2.19). Proper CS are assumed to be multiplied such that the H-momentum becomes $(-1,1,1)$ mod $(12,3,12)$. We set $\langle \mathrm{CS}\rangle = 1$.

Singlet exotics of table 9 are vectorlike.

We find that fields with non-vanishing $\mathrm{U}(1)_6$ quantum numbers are only exotics. This means that $\mathrm{U}(1)_6$ cannot be broken by VEVs of neutral singlets since neutral singlets cannot be exotics. As mentioned before, however, the exactly massless $\mathrm{U}(1)_6$ gauge boson ("exphoton") is still phenomenologically acceptable, since all observable matter fields are neutral under $\mathrm{U}(1)_6$.

In table 10, we present some mass terms of singlet exotics. In this mass table, we tried to combine vectorlike pairs, not listing all off-diagonal terms as before. It would be unwieldy to list all the off-diagonal terms for several tens of singlets. We note that in the above vacuum (2.22), all exotic singlets obtain masses, as can be seen from the pairings listed in table 10. But here it seems that Γ odd fields S_{15} and S_{23} are involved.

States	SM notation	Label
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{3},\frac{-1}{3};\frac{-1}{3},\frac{1}{3},0\right)\left(\frac{-1}{2},\frac{-1}{6},\frac{-2}{3};0^5\right)'$	$\mathbf{1}_{2/3}(T_2^+)$	ξ_1
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{3},\frac{-1}{3};\frac{-1}{3},\frac{1}{3},0\right)\left(\frac{1}{2},\frac{-1}{6},\frac{1}{3};0^5\right)'$	$\mathbf{1}_{2/3}(T_2^+)$	ξ_2
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{-5}{6},\frac{-1}{6},\frac{-1}{2}\right)\left(\frac{1}{2},\frac{-1}{6},\frac{1}{3};0^5\right)'$	$\mathbf{1}_{-1/3}(T_2^+)$	η_1
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{6},\frac{-1}{6},\frac{1}{2}\right)\left(\frac{-1}{2},\frac{5}{6},\frac{1}{3};0^5\right)'$	$\mathbf{1}_{-1/3}(T_2^+)$	η_2
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{6},\frac{-1}{6},\frac{1}{2}\right)\left(\frac{-1}{2},\frac{-1}{6},\frac{-2}{3};0^5\right)'$	$\mathbf{1}_{-1/3}(T_2^+)$	η_3
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{6},\frac{-1}{6},\frac{1}{2}\right)\left(\frac{1}{2},\frac{-1}{6},\frac{1}{3};0^5\right)'$	$2\cdot\mathbf{1}_{-1/3}(T_2^+)$	η_4,η_5
$\left(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3};\frac{1}{3},\frac{1}{3};0,\frac{1}{3},\frac{1}{3}\right)\left(\frac{-1}{2},\frac{1}{6},\frac{-1}{3};0^5\right)'$	$\mathbf{1}_{-2/3}(T_2^-)$	$\overline{\xi}_3$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{2},\frac{-1}{6},\frac{5}{6}\right)\left(\frac{-1}{2},\frac{1}{6},\frac{-1}{3};0^5\right)'$	$\mathbf{1}_{1/3}(T_2^-)$	$\overline{\eta}_6$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{2},\frac{5}{6},\frac{-1}{6}\right)\left(\frac{-1}{2},\frac{1}{6},\frac{-1}{3};0^5\right)'$	$\mathbf{1}_{1/3}(T_2^-)$	$\overline{\eta}_7$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{1}{2},\frac{-1}{6},\frac{-1}{6}\right)\left(\frac{1}{2},\frac{1}{6},\frac{2}{3};0^5\right)'$	$\mathbf{1}_{1/3}(T_2^-)$	$\overline{\eta}_8$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{1}{2},\frac{-1}{6},\frac{-1}{6}\right)\left(\frac{-1}{2},\frac{1}{6},\frac{-1}{3};0^5\right)'$	$2\cdot\mathbf{1}_{1/3}(T_2^-)$	$\overline{\eta}_9,\overline{\eta}_{10}$
$\left(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3};\frac{1}{3},\frac{1}{3};\frac{-2}{3},\frac{-1}{3},0\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	$2\cdot\mathbf{1}_{-2/3}(T_4^+)$	$\overline{\xi}_4$
$\left(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3};\frac{1}{3},\frac{1}{3};\frac{1}{3},\frac{2}{3},0\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	$2\cdot\mathbf{1}_{-2/3}(T_4^+)$	$\overline{\xi}_5$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{6},\frac{-5}{6},\frac{-1}{2}\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	$3\cdot\mathbf{1}_{1/3}(T_4^+)$	$\overline{\eta}_{11}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{5}{6},\frac{1}{6},\frac{-1}{2}\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	$2\cdot\mathbf{1}_{1/3}(T_4^+)$	$\overline{\eta}_{12}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{6},\frac{1}{6},\frac{1}{2}\right)\left(0,\frac{2}{3},\frac{2}{3};0^5\right)'$	$2\cdot\mathbf{1}_{1/3}(T_4^+)$	$\overline{\eta}_{13}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{6},\frac{1}{6},\frac{1}{2}\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	$6\cdot\mathbf{1}_{1/3}(T_4^+)$	$\overline{\eta}_{14},\overline{\eta}_{15},\overline{\eta}_{16}$
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{3},\frac{-1}{3};0,\frac{2}{3},\frac{-1}{3}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	$2\cdot\mathbf{1}_{2/3}(T_4^-)$	ξ_6
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{3},\frac{-1}{3};0,\frac{-1}{3},\frac{2}{3}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	$2\cdot\mathbf{1}_{2/3}(T_4^-)$	ξ_7
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{-1}{2},\frac{-5}{6},\frac{1}{6}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	$3\cdot\mathbf{1}_{-1/3}(T_4^-)$	η_{17}
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{-1}{2},\frac{1}{6},\frac{-5}{6}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	$3\cdot\mathbf{1}_{-1/3}(T_4^-)$	η_{18}
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{2},\frac{1}{6},\frac{1}{6}\right)\left(0,\frac{-2}{3},\frac{-2}{3};0^5\right)'$	$2\cdot\mathbf{1}_{-1/3}(T_4^-)$	η_{19}
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{2},\frac{1}{6},\frac{1}{6}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	$6\cdot\mathbf{1}_{-1/3}(T_4^-)$	$\eta_{20},\eta_{21},\eta_{22}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{7}{12},\frac{7}{12};\frac{1}{12},\frac{-7}{12},\frac{-3}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	$\mathbf{1}_{-2/3}(T_1^+)$	$\overline{\xi}_8$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{-5}{12},\frac{-5}{12};\frac{1}{12},\frac{5}{12},\frac{9}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	$\mathbf{1}_{1/3}(T_1^+)$	$\overline{\eta}_{23}$
$\left(\frac{5}{12},\frac{5}{12},\frac{5}{12};\frac{1}{12},\frac{1}{12};\frac{7}{12},\frac{-1}{12},\frac{3}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	$\mathbf{1}_{1/3}(T_1^+)$	$\overline{\eta}_{24}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{-5}{12},\frac{-5}{12};\frac{1}{12},\frac{-7}{12},\frac{-3}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	$\mathbf{1}_{1/3}(T_1^+)$	$\overline{\eta}_{25}$
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\frac{5}{12},\frac{5}{12};\frac{-9}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{3}{12},\frac{1}{12},\frac{-8}{12};0^5\right)'$	$\mathbf{1}_{-1/3}(T_1^-)$	η_{26}
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{7}{12},\frac{7}{12};\frac{1}{12},\frac{5}{12},\frac{-3}{12}\right)\left(\frac{-3}{12},\frac{-1}{12},\frac{8}{12};0^5\right)'$	$\mathbf{1}_{-2/3}(T_7^+)$	$\overline{\xi}_9$
$\left(\frac{5}{12},\frac{5}{12},\frac{5}{12};\frac{1}{12},\frac{1}{12};\frac{-5}{12},\frac{-1}{12},\frac{3}{12}\right)\left(\frac{9}{12},\frac{-1}{12},\frac{-4}{12};0^5\right)'$	$\mathbf{1}_{1/3}(T_7^+)$	$\overline{\eta}_{27}$
$\left(\frac{5}{12},\frac{5}{12},\frac{5}{12};\frac{1}{12},\frac{1}{12};\frac{-5}{12},\frac{-1}{12},\frac{3}{12}\right)\left(\frac{-3}{12},\frac{-1}{12},\frac{8}{12};0^5\right)'$	$\mathbf{1}_{1/3}(T_7^+)$	$\overline{\eta}_{28}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{-5}{12},\frac{-5}{12};\frac{1}{12},\frac{5}{12},\frac{-3}{12}\right)\left(\frac{9}{12},\frac{-1}{12},\frac{-4}{12};0^5\right)'$	$\mathbf{1}_{1/3}(T_7^+)$	$\overline{\eta}_{29}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{-5}{12},\frac{-5}{12};\frac{1}{12},\frac{5}{12},\frac{-3}{12}\right)\left(\frac{-3}{12},\frac{-1}{12},\frac{8}{12};0^5\right)'$	$\mathbf{1}_{1/3}(T_7^+)$	$\overline{\eta}_{30}$
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\frac{-7}{12},\frac{-7}{12};\frac{3}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{-3}{12},\frac{-5}{12},\frac{4}{12};0^5\right)'$	$\mathbf{1}_{2/3}(T_7^-)$	ξ_{10}
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\frac{5}{12},\frac{5}{12};\frac{3}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{9}{12},\frac{7}{12},\frac{4}{12};0^5\right)'$	$\mathbf{1}_{-1/3}(T_7^-)$	η_{31}
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\frac{5}{12},\frac{5}{12};\frac{3}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{-3}{12},\frac{7}{12},\frac{-8}{12};0^5\right)'$	$\mathbf{1}_{-1/3}(T_7^-)$	η_{32}
$\left(\frac{-5}{12},\frac{-5}{12},\frac{-5}{12};\frac{-1}{12},\frac{-1}{12};\frac{-3}{12},\frac{5}{12},\frac{5}{12}\right)\left(\frac{-3}{12},\frac{-5}{12},\frac{4}{12};0^5\right)'$	$\mathbf{1}_{-1/3}(T_7^-)$	η_{33}
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\frac{5}{12},\frac{5}{12};\frac{3}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{-3}{12},\frac{-5}{12},\frac{4}{12};0^5\right)'$	$2\cdot\mathbf{1}_{-1/3}(T_7^-)$	η_{34},η_{35}

Table 9: Singlet exotics.

4. D and F flat directions

4.1 Anomalous U(1) and D flat directions

There are eight U(1) symmetries in this model. If there is an anomalous U(1), some of the gauge symmetries are broken via the Fayet-Iliopoulos D-term. Indeed, our model has an anomalous $\mathrm{U(1)_A}$ whose charge is given in terms of the original eight U(1) charges as

$$Q_A = 24Y - 30(B-L) + Q_1 + Q_2 + Q_3 + Q_4 - Q_5. \tag{4.1}$$

Pairs	Masses (×proper CS)
$1 \times \{\xi_1(T_2^+),\ \overline{\xi}_9(T_7^+)\}$	$S_7S_9\boxed{S_{23}}$
$1 \times \{\xi_2(T_2^+),\ \overline{\xi}_3(T_2^-)\}$	S_1S_{10}
$1 \times \{\xi_{10}(T_7^-),\ \overline{\xi}_8(T_1^+)\}$	$S_8,\ S_2S_4$
$2 \times \{\xi_6(T_4^-),\ \overline{\xi}_4(T_4^+)\}$	$S_9,\ S_3S_4$
$2 \times \{\xi_7(T_4^-),\ \overline{\xi}_5(T_4^+)\}$	$S_7,\ S_1S_5$
$1 \times \{\eta_1(T_2^+),\ \overline{\eta}_6(T_2^-)\}$	S_4S_{10}
$1 \times \{\eta_2(T_2^+),\ \overline{\eta}_7(T_2^-)\}$	$S_1S_4S_9$
$1 \times \{\eta_3(T_2^+),\ \overline{\eta}_8(T_2^-)\}$	S_5S_{11}
$\{\eta_{4,5}(T_2^+),\ \overline{\eta}_{9,10}(T_2^-)\}$	S_5S_{11}
$2 \times \{\eta_{17}(T_4^-),\ \overline{\eta}_{12}(T_4^+)\}$	$S_8,\ S_2S_4$
$1 \times \{\eta_{17}(T_4^-),\ \overline{\eta}_{27}(T_7^+)\}$	S_6S_{29}
$3 \times \{\eta_{18}(T_4^-),\ \overline{\eta}_{11}(T_4^+)\}$	$(S_4S_{12})^2$
$2 \times \{\eta_{19}(T_4^-),\ \overline{\eta}_{13}(T_4^+)\}$	$S_7,\ S_1S_5$
$\{\eta_{20,21,22}(T_4^-),\ \overline{\eta}_{14,15,16}(T_4^+)\}$	$S_7,\ S_1S_5$
$1 \times \{\eta_{26}(T_1^-),\ \overline{\eta}_{30}(T_7^+)\}$	$S_9,\ S_3S_3$
$1 \times \{\eta_{31}(T_7^-),\ \overline{\eta}_{29}(T_7^+)\}$	$S_1S_7S_{12}\boxed{S_{15}}\ S_{29}$
$1 \times \{\eta_{32}(T_7^-),\ \overline{\eta}_{28}(T_7^+)\}$	$S_{13}\boxed{S_{23}}\ S_{29}$
$1 \times \{\eta_{33}(T_7^-),\ \overline{\eta}_{24}(T_1^+)\}$	$S_7,\ S_1S_5$
$1 \times \{\eta_{34}(T_7^-),\ \overline{\eta}_{23}(T_1^+)\}$	$S_7,\ S_1S_5$
$1 \times \{\eta_{35}(T_7^-),\ \overline{\eta}_{25}(T_1^+)\}$	$S_8,\ S_2S_4$

Table 10: Mass terms for singlet exotics. CS are products of singlet fields given in eq. (2.19). Proper CS are assumed to be multiplied such that the H-momentum becomes $(-1,1,1)$ mod $(12,3,12)$. We set $\langle \mathrm{CS} \rangle = 1$.

The Fayet-Iliopoulos D-term is

$$D^A = \frac{2g}{192\pi^2}\mathrm{Tr}Q_A + \sum_i Q_A(i)\phi^*(i)\phi(i). \tag{4.2}$$

As shown in appendix B, $\mathrm{Tr}Q_A$ is negative, -50. For supersymmetry, the chosen vacuum must satisfy $\langle D^A \rangle = 0$. Thus the summation $\sum_i Q_A(i)\phi^*(i)\phi(i)$ for the nonzero VEVs given in (2.22) should be positive. The VEVs in D^A term potential can break a U(1) at the SUSY minimum. To see how the remaining six U(1)s behave, in table 11 we list the U(1) charges of those singlets with non-vanishing VEVs. The D-flatness conditions for the remaining anomaly free $\mathrm{U(1)_g}$ are

$$\langle D^{(g)} \rangle = \left\langle \sum_i Q_g(i)\phi^*(i)\phi(i) \right\rangle = 0, \quad g = Y,\ a,\ b,\ \cdots,\ e,\ 6. \tag{4.3}$$

One could find the solution to $D^A = D^{(g)} = 0$ $(g = Y,\ a,\ b,\ \cdots,\ e,\ 6)$:

$$\begin{aligned} |S_0|^2 =\ & |S_4|^2 - 2|S_5|^2 - |S_6|^2 - 7|S_7|^2 + 3|S_8|^2 + 3|S_9|^2 \\ & - |S_{10}|^2 + |S_{11}|^2 + |S_{12}|^2 - |S_{13}|^2 + |S_{23}|^2 - \frac{7X^2}{1480}, \end{aligned} \tag{4.4}$$

Label	$\mathcal{P}(f_0)$	Q_Y	Q_A	Q_a	Q_b	Q_c	Q_d	Q_e	Q_6
$S_0(U_2)$	1	0	0	2	6	0	0	0	0
$S_1(T_2^0)$	1	0	$\frac{-4}{3}$	0	$\frac{-4}{3}$	$\frac{-19}{3}$	$\frac{-19}{3}$	$\frac{-4}{3}$	0
$S_2(T_2^0)$	1	0	$\frac{-4}{3}$	-2	$\frac{-10}{3}$	$\frac{11}{3}$	$\frac{11}{3}$	$\frac{-4}{3}$	0
$S_3(T_2^0)$	1	0	$\frac{-4}{3}$	2	$\frac{-10}{3}$	$\frac{11}{3}$	$\frac{11}{3}$	$\frac{-4}{3}$	0
$S_4(T_2^0)$	1+1	0	$\frac{8}{3}$	0	$\frac{8}{3}$	$\frac{-7}{3}$	$\frac{-7}{3}$	$\frac{8}{3}$	0
$S_5(T_2^0)$	1+1	0	$\frac{-4}{3}$	0	$\frac{8}{3}$	$\frac{11}{3}$	$\frac{11}{3}$	$\frac{-4}{3}$	0
$S_6(T_4^0)$	2	0	$\frac{4}{3}$	0	$\frac{16}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	0
$S_7(T_4^0)$	2+3+2	0	$\frac{-8}{3}$	0	$\frac{4}{3}$	$\frac{-8}{3}$	$\frac{-8}{3}$	$\frac{-8}{3}$	0
$S_8(T_4^0)$	2+2+2	0	$\frac{4}{3}$	-2	$\frac{-2}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	0
$S_9(T_4^0)$	2+2+2	0	$\frac{4}{3}$	2	$\frac{-2}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	0
$S_{10}(T_6)$	2	0	0	2	2	5	5	0	0
$S_{11}(T_6)$	2	0	0	-2	-2	-5	-5	0	0
$S_{12}(T_6)$	2	0	0	0	-4	5	5	0	0
$S_{13}(T_6)$	2	0	0	0	4	-5	-5	0	0
$S_{15}(T_1^0)$	1	0	$\frac{-85}{6}$	0	$\frac{4}{3}$	$\frac{19}{3}$	$\frac{-5}{3}$	$\frac{5}{4}$	0
$S_{23}(T_7^0)$	1	0	$\frac{101}{6}$	0	$\frac{-8}{3}$	$\frac{-11}{3}$	$\frac{13}{3}$	$\frac{17}{12}$	0
$S_{29}(T_9)$	2	0	$\frac{31}{2}$	0	0	6	-2	$\frac{1}{12}$	0

Table 11: U(1) charges of scalars developing nonzero VEVs.

$$|S_1|^2 = |S_4|^2 + |S_6|^2 - 7|S_7|^2 + 3|S_8|^2 + 3|S_9|^2 + |S_{10}|^2 - |S_{11}|^2 + |S_{12}|^2 - |S_{13}|^2 + |S_{23}|^2 - \frac{17X^2}{1480}, \tag{4.5}$$

$$|S_2|^2 = 2|S_4|^2 - 2|S_5|^2 - 7|S_7|^2 + 6|S_9|^2 + |S_{23}|^2 - \frac{2X^2}{1480}, \tag{4.6}$$

$$|S_3|^2 = |S_4|^2 + |S_6|^2 + 3|S_8|^2 - 3|S_9|^2 - |S_{10}|^2 + |S_{11}|^2 - |S_{12}|^2 + |S_{13}|^2 - \frac{9X^2}{1480}, \tag{4.7}$$

$$|S_{15}|^2 = |S_{23}|^2 - \frac{6X^2}{185} \tag{4.8}$$

$$|S_{29}|^2 = \frac{3X^2}{185}, \tag{4.9}$$

where $X^2 \equiv \frac{-2g}{192\pi^2}\mathrm{Tr}Q_A$. eq. (4.9) dictates $S_{29} \sim \mathcal{O}(M_{\mathrm{string}}/100)$. The following hierarchical assumption for the VEVs could be consistent with eq. (4.5), (4.6), and (4.7):

$$\frac{1}{2}|S_3|^2 \approx |S_6|^2 \approx |S_{11}|^2 \gtrsim \text{ others.} \tag{4.10}$$

As we mentioned before $U(1)_6$ remains unbroken since there is no neutral singlet carrying a nonzero Q_6. Thus, in addition to photon there exists another strictly massless $U(1)_6$ gauge boson (*exphoton*). It couples only to superheavy exotic matter.

4.2 F flat directions

The neutral singlets in table 2 classified to the five categories as shown in table 12. The

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Classes	Γ'	VEV	Neutral singlets
I	0	non-zero	$S_0,\ S_1,\ S_2,\ \cdots,\ S_{13},\ S_{15},\ S_{23}$
II	$+1$	zero	$S_{14},\ S_{19},\ S_{21},\ S_{25},\ S_{27},\ S_{28}$
III	-1	(non-)zero	$S_{17},\ S_{22},\ S_{24},\ S_{26}$
IV	0	(non-)zero	$S_{16},\ S_{18},\ S_{20}$
V	-1	non-zero	S_{29}

Table 12: Five classes of the neutral singlets.

singlets included in Class I, which do not carry $U(1)_{\Gamma'}$ charges defined in eq. (2.23), are assumed to develop VEVs. The singlets in Class V are also assumed to get VEVs, but they carry the $U(1)_{\Gamma}$ charges of -1. On the other hand, the singlet states in Classes II, III, and IV which carry $\Gamma' = \pm 1$ or 0, do not obtain VEVs.

We note that the R-parity violating operators, $u^c d^c d^c$, QLd^c, LLe^c carry $\Gamma' = -1$. Thus, if VEVs by singlets carrying positive Γ' charges are absent, as in our case, the trilinear R-parity violating terms could not be induced in the superpotential. Hence, if necessary, the singlets in III and IV, which all have the zero or negative Γ' charges, can be allowed to get VEVs. In this paper, however, for simplicity we consider only a vacuum where all singlets in the classes III and IV do not obtain VEVs.

There exist superpotential terms constructed purely with the neutral singlet fields in the class I:

$$\begin{aligned} W = {} & S_1S_6S_{12} + S_3S_6S_{11} + S_1S_8S_{10} + S_3S_8S_{13} + S_2S_9S_{13} + S_4S_7S_{12} \\ & + S_5S_9S_{11} + S_7S_8S_9 + S_7S_{15}S_{23} + S_{10}S_{11} + S_{12}S_{13} + \cdots , \end{aligned} \tag{4.11}$$

where proper CS are assumed to be multiplied. As seen in eq. (2.20), CS are constructed also with the singlets in Class I. In the $\mathbf{Z}_{12}$ orbifold compactification, if a superpotential term w satisfies all the selections rules, then w^{12n+1} $(n = 1, 2, 3, \cdots)$ also does. By including the higher dimensional terms w^{13}, w^{25}, $w^{37}, \cdots$, one can find a vacuum where the singlets of interest develop VEVs of string scale, preserving the F flatness conditions [10]. Moreover, one can always find a re-scaling transformation for the VEVs, leaving intact the F flatness conditions. Using this transformation, one can be consistent also the D flatness conditions can be consistent [10, 19]. With this justification we assume that all the neutral singlets of the class I achieve VEVs of order $M_{\rm string}$ on a vacuum. As argued earlier, the selection rule eq. (2.17) reduces to eq. (2.21) on such a vacuum.

Yukawa couplings containing two or more singlets with zero VEVs are trivial in satisfying the F-flatness conditions. Thus, the couplings, in which two singlets or more from II, III or IV are involved, do not provide non-trivial constraints for F-flatness. However, in the presence of a coupling including only one singlet with vanishing VEV, F-flatness may not be present unless there are more than two such terms.

In the superpotential, the singlets should couple to other fields such that Yukawa couplings are neutral under $U(1)_{\Gamma'}$ and also the other $U(1)$ gauge symmetries: Γ' charges of singlets in the class III should be compensated by being coupled with those of singlets in II. Since all the singlets in II and III do not get VEVs, the couplings between II and III do not provide non-trivial constraints for F-flatness. On the other hand, we should be careful for the couplings between singlets in I and IV, and in II and V, because in these cases couplings only one singlet with a vanishing VEV are possible. In this model, indeed, one can find two or more allowed superpotential terms for each singlet in II. Therefore, the F-flatness conditions, $\partial W/\partial S_{14} = \partial W/\partial S_{16} = \partial W/\partial S_{18} = \cdots = \partial W/\partial S_{28} = 0$ can be satisfied. D-flatness conditions can be satisfied by re-scaling of VEVs. However, in order to get $\langle S_{14}\rangle = \langle S_{16}\rangle = \langle S_{18}\rangle = \cdots = \langle S_{28}\rangle = 0$ as a F-flatness solution and also a μ solution, many F-flatness conditions should turn out to be not independent ones.[4]

5. Vacuum with effective R parity

For the R-parity to be exact, it must be a subgroup of a U(1) gauge group, i.e. it must be a discrete gauge symmetry [16], otherwise large gravitational corrections such as through wormhole processes may violate it. Here, we can include the anomalous U(1) gauge symmetry in string compactification [20], since the matter anomaly is cancelled by the Green-Schwarz mechanism [21]. Taking out the SM nonabelian gauge groups from the E_8 sector leaves five U(1)s among which $U(1)_Y$ cannot be used for the R-parity. Thus, for the R-parity, we are left with four possibilities,

$$\begin{array}{rl} (B-L) &= (\ \frac{2}{3}\ \ \frac{2}{3}\ \ \frac{2}{3}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ )\ (0^8)' \\ X &= (\ -2\ -2\ -2\ -2\ -2\ \ 0\ \ 0\ \ 0\ )\ (0^8)' \\ Q_1 &= (\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ +2\ \ 0\ \ 0\ )\ (0^8)' \\ Q_2 &= (\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ +2\ \ 0\ )\ (0^8)' \end{array} \tag{5.1}$$

For example, another U(1) charge $(-2,-2,-2,-2,-2,-2,-2,0)(0^8)'$ is the linear combination $X - Q_1 - Q_2$. For an R-parity, we can use any odd number of U(1)s given in eq. (5.1). The reason is the following. It is customarily assumed that the SO(10) subgroup of E_8 allows the spinor representation of SO(10). If it arises in the untwisted sector, it must be of the form

$$([\underline{+++++}];\underline{[+++]}) \tag{5.2}$$

where $\pm$ are $\pm\frac{1}{2}$, and the underline means all possible permutations and [ ] means even numbers of sign flips among entries inside the bracket. For the representation (5.2), the U(1) charges of (5.1) are odd. On the other hand, the Higgs doublets in SO(10) have the form

$$(0\ 0\ 0\ \underline{\pm 1\ 0};\pm 1\ 0) \tag{5.3}$$

which give even numbers of the U(1) charges of (5.1). We can define a good R-parity if all the scalar fields developing VEVs carry even numbers of a U(1) charge, say Γ, a linear combination of (5.1). Here, a conflict arises if the phenomenologically needed VEVs require for some Γ odd fields to develop VEVs. Then, in general an exact parity cannot be defined.

[4] In general, all neutral singlets can develop VEVs with the F-flat and D-flat conditions satisfied [10]. Here, one can simply assume VEVs of $S_{14}, S_{16}, \cdots, S_{28}$ are small.

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Let us note possible superpotential terms in the MSSM, generating $\Delta B \neq 0$ operators,

$$d = 4: \quad u^c d^c d^c, \tag{5.4}$$

$$d = 5: \quad QQQL, \quad u^c u^c d^c e^c \tag{5.5}$$

where Q and L are quark and lepton doublets, respectively. The dimension-4 operator of eq. (5.4) alone does not lead to proton decay, but that term together with the $\Delta L \neq 0$ superpotential QLd^c leads to a very fast proton decay and the product of their couplings must satisfy a very stringent constraint, $< 10^{-26}$. The $d = 5$ operators in (5.5) are not that much dangerous, but still the couplings must satisfy constraints, $< 10^{-7}$ [22, 14]. Thus, our prime objective of introducing the R-parity is to forbid $u^c d^c d^c$ up to a sufficiently high level.

A $\mathbf{Z}_2$ subgroup of a U(1) gauge symmetry is welcome for a definition of R-parity. The continuous global U(1) symmetry, being broken by superpotential terms, is not good for an R-parity. For this, we note that the $\mathbf{Z}_2$ subgroup of the $\mathrm{U(1)_X}$ gauge group distinguishes the spinor or the vector origin of our spectrum where

$$X = (-2, -2, -2, -2, -2, \ 0, \ 0, \ 0)(0^8)'. \tag{5.6}$$

For distinguishing two kinds of parity quantum numbers in our model, actually we have a better U(1) gauge symmetry, $\mathrm{U(1)_\Gamma}$, whose generator is

$$\Gamma = X - (Q_2 + Q_3) + \frac{1}{4}(Q_4 + Q_5) + 6(B - L) \tag{5.7}$$

where

$$Q_2 = (0^5; 0, 2, 0)(0^8)', \quad Q_3 = (0^5; 0, 0, 2)(0^8)' \tag{5.8}$$

$$Q_4 = (0^8)(2, 0, 0^6)', \qquad Q_5 = (0^8)(0, 2, 0^6)' \tag{5.9}$$

$$B - L = (\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0^5)(0^8)'\,. \tag{5.10}$$

Q_4 and Q_5 in (5.7) affect only the hidden E_8'. In eq. (5.7), there is an odd number of operators of eq. (5.1), and hence Γ is good for defining a parity. The Γ quantum numbers of standard charge particles are listed in tables 1 and 2. Let us define the R-parity by giving VEVs to some $\Gamma = \pm 2, 0$ neutral singlets,

$$\mathrm{U(1)_\Gamma} \longrightarrow \mathrm{Z_2} \equiv \mathrm{P}. \tag{5.11}$$

The parity defined in this way is multiplicative. Then, the even integer fields carry $P = +1$ and the odd integer fields carry $P = -1$. The P allowed couplings must have the *total* $P = +1$. A more restrictive condition is the $\mathrm{U(1)_\Gamma}$ gauge invariance of couplings: $\sum_i \Gamma(z_i) = 0$, which must be satisfied for the coupling to be present in the original theory.

Inspecting the Γ quantum numbers in the tables, we find that the following fields are possessing 'strange' Γs in defining the R-parity:

$$\overline{D}(T_4^0), \quad D(T_4^0), \quad \overline{D}(T_6), \quad D(T_6), \tag{5.12}$$

$$S_{15}, \quad S_{16}, \quad S_{18}, \quad S_{20}, \quad S_{23} \tag{5.13}$$

which are boxed in tables and 1 and 2. Fields in (5.12) carry the familiar charge $Q_{\rm em}= -\frac{1}{3}$ down type charge but carry even Γs, $Q_{\rm em}=0$ neutral singlets in (5.13) are the familiar neutral Higgs fields but they carry odd Γs. To have an exact R-parity, neutral singlets S_{15}, S_{16}, S_{18}, S_{20}, and S_{23} (the boxed ones) in table 2 should not develop VEVs. But then, the leftover pair in table 4 cannot obtain mass since $\overline{D}(T_4^0)$ carries $P=2$ and $D(T_9)$ carries $P=-1$. To give them mass, some of S_{15}, S_{16}, S_{18}, S_{20}, and S_{23} should develop VEV(s). These VEVs violate the R-parity, i.e. P. So, in our model R-parity violation is inevitable to give large masses to exotics.

5.1 R parity violation

As mentioned above, the dimension-5 operators of the form $QQQL$ and $u^cu^cd^ce^c$, allowed by R-parity, are known to be safe for the proton lifetime constraint in string compactification models [13]. To constrain the R-parity violation from the $\Delta B \neq 0$ processes, therefore, we focus on dimension-4 superpotential terms of the form $u^cd^cd^c$ attached with some of S_{15}, S_{16}, S_{18}, S_{20}, and S_{23}. If there does not exist any such term, the R-parity violation is safe from the proton lifetime bound. The mixing of $\overline{D}(T_4^0)$ with d^c is $O(10^{-16})$ for $m_{\overline{D}(T_4^0)} = O(10^{16})$ GeV, and hence we will not consider the R-parity preserving coupling, $u^cd^c\overline{D}(T_4^0)$.

To study the non-renormalizable couplings, we need products of singlets having non-vanishing VEVs, shown in eq. (2.22). Among these, non-vanishing Γs are carried by $S_0(\Gamma=2), S_{15}(T_1^0,\Gamma=-1)$, and $S_{23}(T_7^0,\Gamma=-1), S_{29}(T_9,\Gamma=-2)$. Since $u^cd^cd^c$ carries $\Gamma=-3$, we need singlet products having $\Gamma=+3$. So we must satisfy two conditions: inclusion of S_0 and inclusion of an odd number of S_{15} and S_{23}. Of course, the H-momentum rules and the gauge invariance conditions must be satisfied. Let us consider the following example of $\Gamma=3$,

$$2S_0 \;\times\; S_{15} \;\times\; \text{any number of } \{S_1,\cdots,S_{13}\}. \qquad (5.14)$$

Eq. (5.14) contains two U_2 fields and one T_1^0 field. With one T_1^0, however, we cannot satisfy the modular invariance condition, eq. (2.18), since all fields in $\{S_1,\cdots,S_{13}\}$ are even twisted. So, the form (5.14) is not allowed. A similar conclusion is drawn if we replace S_{15} by S_{23} in eq. (5.14). Even if $\langle S_{15}\rangle \neq 0$ and $\langle S_{23}\rangle \neq 0$, therefore, the coupling $u^cd^cd^c$ is not generated to all orders.

Actually, there is a simpler argument for the absence of dimension 4 operators such as $u^cd^cd^c$. It comes from the $\mathrm{U}(1)_{\Gamma'}$ conservation. $u^cd^cd^c$ (and also QLd^c, LLe^c) carries $\Gamma'=-1$, and the neutral singlets having VEVs do not carry S with positive Γ'. So, $u^cd^cd^c$ is forbidden to all orders.

However by $\langle S_{15}\rangle \neq 0$ and $\langle S_{23}\rangle \neq 0$, d^c and $\overline{D}(T_4^0,T_6)$ can mix. Eventually, this kind of mixing violates the R-parity. But the violation will be suppressed by

$$O\left(\frac{m_b}{m_D}\right) \sim 10^{-16}.$$

A similar analysis can be done for $\Delta B=0, \Delta L\neq 0$ and R conserving operator $\overline{D}(T_4^0,T_6)QL$. Since proton decay with dimesion 4 operators needs both of $u^cd^cd^c$ and

$\overline{D}(T_4^0, T_6)QL$, we will have the following suppression factor for proton decay operator,

$$O\left(\frac{m_b}{m_D}\right)^2 \sim 10^{-32} \tag{5.15}$$

which is completely negligible. Then, proton decay proceeds dominantly by the dimension 5 operators [22]. Being an SSM, gauge boson exchanges do not lead to proton decay. But it is not clear whether $p \to e^+ + (K, \overline{K})^0$ dominates over $p \to e^+ + \pi^0$ since there is no reason that $d = 5$ non-renormalizable couplings are flavor distinguished.

5.2 Effective R parity of light particles and CDM candidate

The observation that the modular invariance condition removes the coupling of the form (5.14) hints that there might be an effective R-parity among light (electroweak scale) particles. It arises from the fact that the odd R singlets of table 2 are in odd twisted sectors, and we need odd number of these odd twisted sector VEVs to have R-parity violating couplings. But the odd number of twisted sectors cannot make modular invariant Yukawa couplings since the other non-vanishing VEVs are carried by the fields in the even twisted sectors.

ν^c in eq. (2.13) can obtain a large mass by singlet VEVs, and considered to be in the intermediate scale. We consider $H_u(U_2)$ and $H_d(U_2)$ are the electroweak scale Higgs doublets. All the other vectorlike pairs in table 1 are considered to be at the string scale. Thus, the light particles of table 1 are Q, d^c, u^c, L, e^c of eq. (2.13), which carry $P = 1$. If we assume that boxed fields in table 2 are superheavy, the light (electroweak scale) Higgs fields, including neutral singlets, carry even P quantum numbers. In this way, we have an effective R-parity among light fields. But the original theory does not respect the R-parity, including all particles. However, this R-parity violation must include heavy particles at the string scale, which is not phenomenologically harmful. Since any R-parity violation among light particles must occur at least with a suppression factor of $O(M_{\rm string})$ for $\Delta B = 0$ and $\Delta L \neq 0$ operators, the lightest supersymmetric particle (LSP) defined among light fields must live at least 10^{22} years, estimated by multiplying $(m_{\rm LSP}/m_p)^5$ to the proton lifetime estimate obtained from dimension 5 operators. Therefore, even though the R-parity is not exact, we have a cold dark matter (CDM) candidate LSP which lives sufficiently long enough.

6. Model without exotics

The VEVs given in eq. (2.22) break $U(1)$gauge symmetries with leaving only (SM gauge group)$\times$[SO(10) $\times$ U(1)$_6$]$'$. Because the SM fields are completely blind to U(1)$'_6$, it is possible to break a linear combination of U(1)$_{\rm em}$ and U(1)$'_6$, leaving only one $U(1)$ unbroken. Let us call this unbroken $U(1)$ the $U(1)$ of quantum electrodynamics, $\tilde{\rm U}(1)_{\rm em}$. We choose the symmetry breaking direction such that there does not appear any exotics, i.e. $\tilde{\rm U}(1)_{\rm em}$ charges of particles are integers for color singlets, $+\frac{2}{3}, -\frac{1}{3}$ for color triplets ($\mathbf{3}$), and $-\frac{2}{3}, +\frac{1}{3}$ for color anti-triplets ($\mathbf{3}^*$). The electroweak hypercharge direction (2.9) fulfils this possibility.

$\tilde{Y}$ from (2.9)	$\mathrm{SU(3)_c \times SU(2)_L}$	Exotics in Model E
$-\frac{1}{3}$	$(\mathbf{3}, \mathbf{1})$	$\alpha_1^0,\ \alpha_3^0,\ 2 \cdot \alpha_4^0$
$+\frac{1}{3}$	$(\mathbf{3}^*, \mathbf{1})$	$\overline{\alpha}_2^0,\ 3 \cdot \overline{\alpha}_5^0$
$-\frac{2}{3}$	$(\mathbf{3}^*, \mathbf{1})$	$2 \cdot \overline{\alpha}_6^{-1/3}$
$+\frac{2}{3}$	$(\mathbf{3}, \mathbf{1})$	$2 \cdot \alpha_7^{1/3}$
$-\frac{1}{2}$	$(\mathbf{1}, \mathbf{2})$	$\overline{\delta}_1,\ \delta_3,\ 3 \cdot \overline{\delta}_4,\ 2 \cdot \overline{\delta}_5$
$\frac{1}{2}$	$(\mathbf{1}, \mathbf{2})$	$\delta_2,\ 2 \cdot \delta_6,\ 2 \cdot \delta_7,\ \overline{\delta}_8,\ \delta_9$
0	$(\mathbf{1}, \mathbf{1})$	$\xi_1,\ \eta_1,\ \eta_2,\ \eta_{4,5},\ \overline{\eta}_6,\ \overline{\eta}_7,\ \overline{\eta}_{9,10},\ 3 \cdot \overline{\eta}_{11},\ 2 \cdot \overline{\eta}_{12},$ $2 \cdot \overline{\eta}_{14,15,16},\ 3 \cdot \eta_{17},\ 3 \cdot \eta_{18},\ 2 \cdot \eta_{20,21,22},$ $\overline{\eta}_{23},\ \overline{\eta}_{24},\ \overline{\eta}_{25},\ \overline{\xi}_9,\ \overline{\eta}_{27},\ \overline{\eta}_{29},\ \eta_{31},\ \eta_{33},\ \eta_{34,35}$
-1	$(\mathbf{1}, \mathbf{1})$	$\eta_3,\ \overline{\xi}_3,\ 2 \cdot \overline{\xi}_4,\ 2 \cdot \overline{\xi}_5,\ 2 \cdot \eta_{19},\ \overline{\xi}_8,\ \eta_{26},\ \eta_{32}$
$+1$	$(\mathbf{1}, \mathbf{1})$	$\xi_2,\ \overline{\eta}_8,\ 2 \cdot \overline{\eta}_{13},\ 2 \cdot \xi_6,\ 2 \cdot \xi_7,\ \overline{\eta}_{28},\ \overline{\eta}_{30},\ \xi_{10}$

Table 13: Model S contains no exotics. Previous exotics carry the standard charges as shown in the first column. The charges of the remaining states in Model S are the same as those in Model E.

This is achieved by giving a VEV(s) to an exotic singlet(s). For instance, let us choose just η_1 and $\overline{\eta}_6$. Both $\langle \eta_1 \overline{\eta}_6 \rangle = 0$ (Model E) and $\langle \eta_1 \overline{\eta}_6 \rangle \neq 0$ (Model S) can be consistent with SUSY, because the superpotential allows $W = \eta_1 \overline{\eta}_6 S_4 S_{10} + (\eta_1 \overline{\eta}_6 S_4 S_{10})^{13} + \cdots$, and both vacua can satisfy the F- and D-flatness conditions. If $\langle \eta_1 \overline{\eta}_6 \rangle \neq 0$, the surviving U(1) gauge symmetry is a linear combination of $\mathrm{U(1)_Y}$ and $\mathrm{U(1)_6}$, i.e. eq. (2.9)

$$\tilde{Y} = Y + \frac{1}{2} Q_6. \tag{6.1}$$

Under this new $\mathrm{U(1)_{\tilde{Y}}}$, all the exotics in Model E carry the regular hypercharges observed in the SSM. With the new $\mathrm{U(1)_{\tilde{Y}}}$, thus, *all the exotics found in Model E are moved into states with the standard charges* as shown in table 13. They still form vectorlike representations under the SM gauge symmetry. Their mass terms discussed in section 3 are still valid. On the other hand, the regularly charged states in Model E, which originate from U, T_3, T_6, T_9 and T_k^0 $(k = 1, 2, 4, 7)$ sectors, are not affected by this addition since they were not charged under $\mathrm{U(1)_6}$ in the beginning. As mentioned below eq. (2.9), the hypercharge operator in Model S gives $\sin^2\theta_W^0 = \frac{3}{14}$ at the string scale. In this case, therefore, more (vectorlike) $\mathrm{SU(3)_c}$ triplets and $\mathrm{SU(2)_L}$ doublets at intermediate mass scales would be needed to explain $\sin^2\theta_W \approx 0.23$ at the electroweak scale. The discussion on the effective R-parity is similar to that of Model E.

In this short section, we observed that models without exotics are possible, but in such models it might be difficult to obtain $\sin^2 \theta_W^0 = \frac{3}{8}$ at the string scale.

7. Conclusions

We have constructed an SSM from a $\mathbf{Z}_{12-I}$ orbifold compactification. In the vacuum chosen in (2.22), we achieve

- An SSM with three families with the third family in the untwisted sector,
- At the string scale, $\sin^2\theta_W^0 = \frac{3}{8}$,
- It is possible to have one pair of light Higgs doublets H_u and H_d from the untwisted sector,
- There exist Yukawa couplings for phenomenologically satisfactory quark and lepton masses,
- All vectorlike color triplets D and $\overline{D}$ obtain masses,
- All exotic particles are vectorlike and obtain masses,
- D- and F-flat directions are possible,
- An effective R-parity (more accurately an effective matter parity), P, can be embedded as a discrete group of gauged $\mathrm{U}(1)_\Gamma$,
- All exotics carry nonzero $\mathrm{U}(1)_6$ quantum numbers,
- $\mathrm{U}(1)_{\mathrm{em}}$ and $\mathrm{U}(1)_6$ are not broken. Therefore, there exist at least two massless color singlet gauge bosons: photon and *exphoton* (meaning the massless gauge boson coupling to exotic particles only).
- If $\mathrm{U}(1)_{\mathrm{em}}$ and $\mathrm{U}(1)_6$ are properly broken to give $\tilde{\mathrm{U}}(1)_{\mathrm{em}}$ unbroken, then one can convert all exotics into states with the standard charges.

In sum we have shown that there exists a very satisfactory string vacuum which meets all phenomenological constraints. At the least, this paper shows the existence proof of the MSSM from superstring. But why the VEVs of eq. (2.22) should be taken as given there is not understood yet in this paper.

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A. Massless Spectrum

The model presented in eq. (2.1) gives

$$V^2 - \phi^2 = 1, \quad a_3^2 = 4, \quad V\cdot a_3 = 1, \tag{A.1}$$
$$V_+^2 - \phi^2 = 7, \quad V_-^2 - \phi^2 = 3. \tag{A.2}$$

Then, the gauge group is

$$\left[\{\mathrm{SU}(3)_{\mathrm{c}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}\} \times \mathrm{U}(1)_{\mathrm{B-L}} \times \mathrm{U}(1)^3\right] \times \left[\mathrm{SO}(10)\times \mathrm{U}(1)^3\right]'. \tag{A.3}$$

$P\cdot V$	Visible States	χ	SM
$\frac{7}{12}$ (U_1)	$(\underline{++-};\underline{+-};+++)$	L	Q
	$(---;\underline{+-};+--)$	L	L
$\frac{4}{12}$ (U_2)	$(0,0,0;\underline{1,0};0,0,1)$	L	H_d
	$(0,0,0;\underline{-1,0};-1,0,0)$	L	H_u
	$(0,0,0;0,0;1,0,-1)$	L	$\mathbf{1_0}\equiv S_0$
$\frac{1}{12}$ (U_3)	$(\underline{+--};--;+++)$	L	d^c
	$(+++;++;+++)$	L	ν^c
	$(\underline{+--};++;+--)$	L	u^c
	$(+++;--;-+-)$	L	e^c

Table 14: Visible sector chiral fields from the U sector. There is no hidden sector chiral fields in the U sector.

$P+6V$	χ	$(N^L)_j$	Θ_0	$\mathcal{P}_6$	SM
$\left(\underline{1,0,0};0,0;0^3\right)\left(\frac{-1}{2},\frac{1}{2};0^6\right)'$	L	0	$\frac{-1}{3}$	3	$3\cdot\overline{D}^{1/3}$
$\left(\underline{-1,0,0};0,0;0^3\right)\left(\frac{1}{2},\frac{-1}{2};0^6\right)'$	L	0	$\frac{-1}{3}$	3	$3\cdot D^{-1/3}$
$\left(0,0,0;\underline{1,0};0^3\right)\left(\frac{1}{2},\frac{-1}{2};0^6\right)'$	L	0	$\frac{1}{6}$	2	$2\cdot H_d$
$\left(0,0,0;\underline{-1,0};0^3;0^3\right)\left(\frac{-1}{2},\frac{1}{2};0^6\right)'$	L	0	$\frac{1}{6}$	2	$2\cdot H_u$
$\left(0^5;1,0,0\right)\left(\frac{-1}{2},\frac{1}{2};0^6\right)'$	L	0	$\frac{-1}{6}$	2	$2\cdot\mathbf{1_0}$
$\left(0^5;-1,0,0\right)\left(\frac{1}{2},\frac{-1}{2};0^6\right)'$	L	0	$\frac{1}{2}$	2	$2\cdot\mathbf{1_0}$
$\left(0^5;0,0,1\right)\left(\frac{-1}{2},\frac{1}{2};0^6\right)'$	L	0	$\frac{1}{2}$	2	$2\cdot\mathbf{1_0}$
$\left(0^5;0,0,-1\right)\left(\frac{1}{2},\frac{-1}{2};0^6\right)'$	L	0	$\frac{-1}{6}$	2	$2\cdot\mathbf{1_0}$

Table 15: Massless states satisfying $(P+6V)\cdot W=0$ mod Z in T_6.

In this model, there are eight U(1) symmetries whose charges are

$$Y = \left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{2},\frac{-1}{2};0^3\right)\left(0^8\right)' \tag{A.4}$$

$$B-L = \left(\frac{2}{3},\frac{2}{3},\frac{2}{3};0^2;0^3\right)\left(0^8\right)' \tag{A.5}$$

$$Q_1 = \left(0^5;2,0,0\right)\left(0^8\right)' \tag{A.6}$$

$$Q_2 = \left(0^5;0,2,0\right)\left(0^8\right)' \tag{A.7}$$

$$Q_3 = \left(0^5;0,0,2\right)\left(0^8\right)' \tag{A.8}$$

$$Q_4 = \left(0^8\right)\left(2,0,0;0^5\right)' \tag{A.9}$$

$$Q_5 = \left(0^8\right)\left(0,2,0;0^5\right)' \tag{A.10}$$

$$Q_6 = \left(0^8\right)\left(0,0,2;0^5\right)'. \tag{A.11}$$

$P+3V$	χ	$(N^L)_j$	$\Theta_{L,R}$	SM [SO(10)′]
$\left(\underline{\tfrac{3}{4},\tfrac{-1}{4},\tfrac{-1}{4}};\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}\right)\left(\tfrac{3}{4},\tfrac{1}{4},0;0^5\right)'$	L	0	$\frac{1}{3}$	$\overline{D}^{1/3}$
$\left(\underline{\tfrac{3}{4},\tfrac{-1}{4},\tfrac{-1}{4}};\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}\right)\left(\tfrac{3}{4},\tfrac{1}{4},0;0^5\right)'$	R	0	0	$2\cdot D^{-1/3\,*}$, or
	L			$(2\cdot D^{-1/3}$ from $T_9)$
$\left(\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4};\underline{\tfrac{3}{4},\tfrac{-1}{4}};\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}\right)\left(\tfrac{-1}{4},\tfrac{-3}{4},0;0^5\right)'$	L	0	$\frac{1}{3}$	H_d
$\left(\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4};\underline{\tfrac{3}{4},\tfrac{-1}{4}};\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}\right)\left(\tfrac{-1}{4},\tfrac{-3}{4},0;0^5\right)'$	R	0	0	$2\cdot H_u^*$, or
	L			$(2\cdot H_u$ from $T_9)$
$\left(\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{-3}{4},\tfrac{1}{4},\tfrac{1}{4}\right)\left(\tfrac{-1}{4},\tfrac{-3}{4},0;0^5\right)'$	L	0	$\frac{-1}{3}$	$\mathbf{1_0}$
$\left(\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{-3}{4},\tfrac{1}{4},\tfrac{1}{4}\right)\left(\tfrac{-1}{4},\tfrac{-3}{4},0;0^5\right)'$	R	0	$\frac{1}{3}$	$\mathbf{1_0}^*$
$\left(\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{1}{4},\tfrac{1}{4},\tfrac{-3}{4}\right)\left(\tfrac{-1}{4},\tfrac{-3}{4},0;0^5\right)'$	L	0	0	$2\cdot\mathbf{1_0}$
$\left(\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{-1}{4},\tfrac{-1}{4};\tfrac{1}{4},\tfrac{1}{4},\tfrac{-3}{4}\right)\left(\tfrac{-1}{4},\tfrac{-3}{4},0;0^5\right)'$	R	0	$\frac{-1}{3}$	$\mathbf{1_0}^*$
$\left(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4};\tfrac{1}{4},\tfrac{1}{4};\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4}\right)\left(\tfrac{3}{4},\tfrac{1}{4},0;0^5\right)'$	L	$1_1, 1_3$	$\frac{1}{3}$	$2\cdot\mathbf{1_0}$
$\left(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4};\tfrac{1}{4},\tfrac{1}{4};\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4}\right)\left(\tfrac{3}{4},\tfrac{1}{4},0;0^5\right)'$	R	$1_1, 1_3$	0	$3\cdot\mathbf{1_0}^*$
$\left(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4};\tfrac{1}{4},\tfrac{1}{4};\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4}\right)\left(\tfrac{-1}{4},\tfrac{1}{4},0;\underline{\pm1,0^4}\right)'$	L	0	0	$2\cdot\mathbf{1_0}$ $[\mathbf{10'}]$
$\left(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4};\tfrac{1}{4},\tfrac{1}{4};\tfrac{-1}{4},\tfrac{-1}{4},\tfrac{-1}{4}\right)\left(\tfrac{-1}{4},\tfrac{1}{4},0;\underline{\pm1,0^4}\right)'$	R	0	$\frac{-1}{3}$	$\mathbf{1_0}^*$ $[\mathbf{10'}]$

Table 16: Massless states satisfying $(P+3V)\cdot W = 0$ mod Z in T_3. The starred chirality R fields in T_3 can be represented also by un-starred chirality L fields with the opposite quantum numbers in T_9, as shown in two lines. There are, in total, three $\mathbf{10'}$s of the hidden SO(10)′ from the T_3 and T_9 sectors. The other states in T_3 and T_9 are singlets under the hidden gauge group. The multiplicity is shown as the coefficient in the last column.

There are two familiar U(1) charges

$$Y = \left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{2},\frac{-1}{2};0,0,0\right)\left(0^8\right)', \tag{A.12}$$

$$Q_{B-L} \equiv B-L = \left(\frac{2}{3},\frac{2}{3},\frac{2}{3};0,0;0,0,0\right)\left(0^8\right)'. \tag{A.13}$$

Note that X of the flipped SU(5) is a combination of $B-L$ and Y,

$$X = (-2,-2,-2,-2,-2,;0,0,0)\left(0^8\right)' = -5(B-L)+4Y. \tag{A.14}$$

The $\mathrm{U}(1)_\Gamma$ charge used in the text is

$$\Gamma = X + \frac{1}{4}(Q_4+Q_5) - (Q_2+Q_3) + 6(B-L). \tag{A.15}$$

Using the technique and notation of [11], massless fields are calculated. In table 14, we list the massless fields from the untwisted sector. There is one singlet S_0 which cannot be a member of the SO(10) spinor. In tables 15 and 16, we list massless fields in T_6 and T_3(and T_9) which are not affected by Wilson lines. In tables 17, 18, 19, and 20, we list massless fields of T_2, T_4, T_1, and T_5 sectors, respectively. For the SM particles, we use the familiar notations: $Q, u^c, d^c, L, e^c, \nu^c$ for sixteen fields of the SM and S for SO(10)′ singlets.

$P+2V$	χ	$(N^L)_j$	$\mathcal{P}_2(f_0)$	SM
$\left(0^5;\frac{-2}{3},\frac{-2}{3},\frac{-1}{3}\right)\left(\frac{1}{2},\frac{-1}{2};0^6\right)'$	L	0	1	$\mathbf{1_0}$
$\left(0^5;\frac{-2}{3},\frac{1}{3},\frac{2}{3}\right)\left(\frac{-1}{2},\frac{1}{2};0^6\right)'$	L	0	1	$\mathbf{1_0}$
$\left(0^5;\frac{1}{3},\frac{-2}{3},\frac{2}{3}\right)\left(\frac{-1}{2},\frac{1}{2};0^6\right)'$	L	0	1	$\mathbf{1_0}$
$\left(0^5;\frac{1}{3},\frac{1}{3},\frac{-1}{3}\right)\left(\frac{1}{2},\frac{-1}{2};0^6\right)'$	L	$2_{\bar{1}},\ 2_3$	$1+1$	$2\cdot\mathbf{1_0}$
$\left(0^5;\frac{1}{3},\frac{1}{3},\frac{-1}{3}\right)\left(\frac{-1}{2},\frac{1}{2};0^6\right)'$	L	$1_{\bar{2}},\ \{1_{\bar{1}}+1_3\}$	$1+1$	$2\cdot\mathbf{1_0}$
$P+2V_+$	χ	$(N^L)_j$	$\mathcal{P}_2(f_+)$	SM
$\left(\underline{\frac{5}{6},\frac{-1}{6},\frac{-1}{6}};\frac{1}{6},\frac{1}{6};\frac{1}{6},\frac{-1}{6},\frac{-1}{2}\right)\left(\frac{1}{2},\frac{-1}{6},\frac{1}{3};0^5\right)'$	L	0	1	$\overline{\alpha}^0$
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{3},\frac{-1}{3};\frac{-1}{3},\frac{1}{3},0\right)\left(\frac{-1}{2},\frac{-1}{6},\frac{-2}{3};0^5\right)'$	L	0	1	$\xi^{2/3}$
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{3},\frac{-1}{3};\frac{-1}{3},\frac{1}{3},0\right)\left(\frac{1}{2},\frac{-1}{6},\frac{1}{3};0^5\right)'$	L	1_3	1	$\xi^{2/3}$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{-5}{6},\frac{-1}{6},\frac{-1}{2}\right)\left(\frac{1}{2},\frac{-1}{6},\frac{1}{3};0^5\right)'$	L	0	1	$\eta^{-1/3}$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{6},\frac{-1}{6},\frac{1}{2}\right)\left(\frac{-1}{2},\frac{5}{6},\frac{1}{3};0^5\right)'$	L	0	1	$\eta^{-1/3}$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{6},\frac{-1}{6},\frac{1}{2}\right)\left(\frac{-1}{2},\frac{-1}{6},\frac{-2}{3};0^5\right)'$	L	1_3	1	$\eta^{-1/3}$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{6},\frac{-1}{6},\frac{1}{2}\right)\left(\frac{1}{2},\frac{-1}{6},\frac{1}{3};0^5\right)'$	L	$2_{\bar{1}},\ 2_3$	$1+1$	$2\cdot\eta^{-1/3}$
$P+2V_-$	χ	$(N^L)_j$	$\mathcal{P}_2(f_-)$	SM
$\left(\underline{\frac{-5}{6},\frac{1}{6},\frac{1}{6}};\frac{-1}{6},\frac{-1}{6};\frac{-1}{2},\frac{-1}{6},\frac{-1}{6}\right)\left(\frac{-1}{2},\frac{1}{6},\frac{-1}{3};0^5\right)'$	L	0	1	α^0
$\left(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3};\frac{1}{3},\frac{1}{3};0,\frac{1}{3},\frac{1}{3}\right)\left(\frac{-1}{2},\frac{1}{6},\frac{-1}{3};0^5\right)'$	L	1_3	1	$\overline{\xi}^{-2/3}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{2},\frac{-1}{6},\frac{5}{6}\right)\left(\frac{-1}{2},\frac{1}{6},\frac{-1}{3};0^5\right)'$	L	0	1	$\overline{\eta}^{1/3}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{2},\frac{5}{6},\frac{-1}{6}\right)\left(\frac{-1}{2},\frac{1}{6},\frac{-1}{3};0^5\right)'$	L	0	1	$\overline{\eta}^{1/3}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{1}{2},\frac{-1}{6},\frac{-1}{6}\right)\left(\frac{1}{2},\frac{1}{6},\frac{2}{3};0^5\right)'$	L	1_3	1	$\overline{\eta}^{1/3}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{1}{2},\frac{-1}{6},\frac{-1}{6}\right)\left(\frac{-1}{2},\frac{1}{6},\frac{-1}{3};0^5\right)'$	L	$1_{\bar{2}},\ \{1_{\bar{1}}+1_3\}$	$1+1$	$2\cdot\overline{\eta}^{1/3}$

Table 17: Chiral matter fields satisfying $\Theta_0 = 0$ in the T_2^0 sector, $\Theta_+ = 0$ in the T_2^+ sector, and $\Theta_- = 0$ in the T_2^- sector.

The Higgs doublets are denoted by H_u and H_d. The color triplets with $Q_{\rm em}= -\frac{1}{3}$, which in principle can mix with d, are denoted as D.

Exotic particles appear in the sectors affected by Wilson lines: $T_2^{\pm}, T_4^{\pm}, T_1^{\pm}$, and $T_5^{\pm}$. For these exotics, we use the following notations:

$$\begin{aligned}
&\alpha_i,\ \overline{\alpha}_j : \text{color exotics } \mathbf{3} \text{ and } \mathbf{3}^* \\
&\delta_i,\ \overline{\delta}_j : \text{SU(2) doublet exotics} \\
&\xi_i,\ \overline{\xi}_j : Q_{\rm em} = \pm\frac{2}{3}\ \text{SU(3)} \times \text{SU(2) singlet exotics} \\
&\eta_i,\ \overline{\eta}_j : Q_{\rm em} = \mp\frac{1}{3}\ \text{SU(3)} \times \text{SU(2) singlet exotics}
\end{aligned} \tag{A.16}$$

If some exotics do not obtain mass, the model must be excluded from phenomenological consideration. In the text, we have shown that all exotics obtain masses. This *massive exotics condition* determines the vacuum where nonvanishing VEVs of S fields are dictated. There are many possibilities for giving masses to exotic particles. In this paper, we chose the minimum number of neutral singlet VEVs, eq. (2.22).

$P+4V$	χ	$(N^L)_j$	Θ_0	$\mathcal{P}_4(f_0)$	SM
$\left(\underline{+--};--;\frac{1}{6},\frac{1}{6},\frac{-1}{6}\right)(0^8)'$	L	0	$\frac{-1}{4}$	2	$2\cdot d^c$
$\left(---;\underline{+-};\frac{1}{6},\frac{1}{6},\frac{-1}{6}\right)(0^8)'$	L	0	$\frac{-1}{4}$	2	$2\cdot L$
$\left(\underline{+--}++;\frac{1}{6},\frac{1}{6},\frac{-1}{6}\right)(0^8)'$	L	0	$\frac{1}{4}$	2	$2\cdot u^c$
$\left(\underline{++-};\underline{+-};\frac{1}{6},\frac{1}{6},\frac{-1}{6}\right)(0^8)'$	L	0	$\frac{1}{4}$	2	$2\cdot Q$
$\left(+++;--;\frac{1}{6},\frac{1}{6},\frac{-1}{6}\right)(0^8)'$	L	0	$\frac{1}{4}$	2	$2\cdot e^c$
$\left(+++;++;\frac{1}{6},\frac{1}{6},\frac{-1}{6}\right)(0^8)'$	L	0	$\frac{-1}{4}$	2	$2\cdot \nu^c$
$\left(\underline{1,0,0};0,0;\frac{-1}{3},\frac{-1}{3},\frac{1}{3}\right)(0^8)'$	L	0	0	3	$3\cdot \overline{D}^{1/3}$
$\left(\underline{-1,0,0};0,0;\frac{-1}{3},\frac{-1}{3},\frac{1}{3}\right)(0^8)'$	L	0	$\frac{1}{2}$	2	$2\cdot D^{-1/3}$
$\left(0,0,0;\underline{1,0};\frac{-1}{3},\frac{-1}{3},\frac{1}{3}\right)(0^8)'$	L	0	0	3	$3\cdot H_d$
$\left(0,0,0;\underline{-1,0};\frac{-1}{3},\frac{-1}{3},\frac{1}{3}\right)(0^8)'$	L	0	$\frac{1}{2}$	2	$2\cdot H_u$
$\left(0,0,0;0,0;\frac{2}{3},\frac{2}{3},\frac{-2}{3}\right)(0^8)'$	L	0	$\frac{1}{2}$	2	$2\cdot \mathbf{1_0}$
$\left(0,0,0;0,0;\frac{-1}{3},\frac{-1}{3},\frac{-2}{3}\right)(0^8)'$	L	$1_{\bar{1}},1_2,1_3$	$\frac{-1}{4},0,\frac{1}{4}$	$2+3+2$	$7\cdot \mathbf{1_0}$
$\left(0,0,0;0,0;\frac{-1}{3},\frac{2}{3},\frac{1}{3}\right)(0^8)'$	L	$1_{\bar{1}},1_2,1_3$	$\frac{1}{4},\frac{1}{2},\frac{-1}{4}$	$2+2+2$	$6\cdot \mathbf{1_0}$
$\left(0,0,0;0,0;\frac{2}{3},\frac{-1}{3},\frac{1}{3}\right)(0^8)'$	L	$1_{\bar{1}},1_2,1_3$	$\frac{1}{4},\frac{1}{2},\frac{-1}{4}$	$2+2+2$	$6\cdot \mathbf{1_0}$
$P+4V_+$	χ	$(N^L)_j$	Θ_+	$\mathcal{P}_4(f_+)$	SM
$\left(\underline{\frac{-5}{6},\frac{1}{6},\frac{1}{6}};\frac{-1}{6},\frac{-1}{6};\frac{-1}{6},\frac{1}{6},\frac{-1}{2}\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	L	0	$\frac{1}{2}$	2	$2\cdot \alpha^0$
$\left(\underline{\frac{2}{3},\frac{-1}{3},\frac{-1}{3}};\frac{1}{3},\frac{1}{3};\frac{1}{3},\frac{-1}{3},0\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	L	0	$\frac{1}{4}$	2	$2\cdot \overline{\alpha}^{-1/3}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\underline{\frac{5}{6},\frac{-1}{6}};\frac{-1}{6},\frac{1}{6},\frac{-1}{2}\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	L	0	0	3	$3\cdot \overline{\delta}^{-1/6}$
$\left(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3};\underline{\frac{-2}{3},\frac{1}{3}};\frac{1}{3},\frac{-1}{3},0\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	L	0	$\frac{-1}{4}$	2	$2\cdot \overline{\delta}^{-1/6}$
$\left(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3};\frac{1}{3},\frac{1}{3};\frac{-2}{3},\frac{-1}{3},0\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	L	0	$\frac{1}{4}$	2	$2\cdot \overline{\xi}^{-2/3}$
$\left(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3};\frac{1}{3},\frac{1}{3};\frac{1}{3},\frac{2}{3},0\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	L	0	$\frac{-1}{4}$	2	$2\cdot \overline{\xi}^{-2/3}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{6},\frac{-5}{6},\frac{-1}{2}\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	L	0	0	3	$3\cdot \overline{\eta}^{1/3}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{5}{6},\frac{1}{6},\frac{-1}{2}\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	L	0	$\frac{1}{2}$	2	$2\cdot \overline{\eta}^{1/3}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{6},\frac{1}{6},\frac{1}{2}\right)\left(0,\frac{2}{3},\frac{2}{3};0^5\right)'$	L	0	$\frac{1}{4}$	2	$2\cdot \overline{\eta}^{1/3}$
$\left(\frac{1}{6},\frac{1}{6},\frac{1}{6};\frac{-1}{6},\frac{-1}{6};\frac{-1}{6},\frac{1}{6},\frac{1}{2}\right)\left(0,\frac{-1}{3},\frac{-1}{3};0^5\right)'$	L	$1_{\bar{1}},1_2,1_3$	$\frac{1}{4},\frac{1}{2},\frac{-1}{4}$	$2+2+2$	$6\cdot \overline{\eta}^{1/3}$
$P+4V_-$	χ	$(N^L)_j$	Θ_-	$\mathcal{P}_4(f_-)$	SM
$\left(\underline{\frac{5}{6},\frac{-1}{6},\frac{-1}{6}};\frac{1}{6},\frac{1}{6};\frac{-1}{2},\frac{1}{6},\frac{1}{6}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	L	0	0	3	$3\cdot \overline{\alpha}^0$
$\left(\underline{\frac{-2}{3},\frac{1}{3},\frac{1}{3}};\frac{-1}{3},\frac{-1}{3};0,\frac{-1}{3},\frac{-1}{3}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	L	0	$\frac{1}{4}$	2	$2\cdot \alpha^{1/3}$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\underline{\frac{-5}{6},\frac{1}{6}};\frac{-1}{2},\frac{1}{6},\frac{1}{6}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	L	0	$\frac{1}{2}$	2	$2\cdot \delta^{1/6}$
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\underline{\frac{2}{3},\frac{-1}{3}};0,\frac{-1}{3},\frac{-1}{3}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	L	0	$\frac{-1}{4}$	2	$2\cdot \delta^{1/6}$
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{3},\frac{-1}{3};0,\frac{2}{3},\frac{-1}{3}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	L	0	$\frac{1}{4}$	2	$2\cdot \xi^{2/3}$
$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{-1}{3},\frac{-1}{3};0,\frac{-1}{3},\frac{2}{3}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	L	0	$\frac{1}{4}$	2	$2\cdot \xi^{2/3}$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{-1}{2},\frac{-5}{6},\frac{1}{6}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	L	0	0	3	$3\cdot \eta^{-1/3}$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{-1}{2},\frac{1}{6},\frac{-5}{6}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	L	0	0	3	$3\cdot \eta^{-1/3}$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{2},\frac{1}{6},\frac{1}{6}\right)\left(0,\frac{-2}{3},\frac{-2}{3};0^5\right)'$	L	0	$\frac{-1}{4}$	2	$2\cdot \eta^{-1/3}$
$\left(\frac{-1}{6},\frac{-1}{6},\frac{-1}{6};\frac{1}{6},\frac{1}{6};\frac{1}{2},\frac{1}{6},\frac{1}{6}\right)\left(0,\frac{1}{3},\frac{1}{3};0^5\right)'$	L	$1_{\bar{1}},1_2,1_3$	$\frac{1}{4},\frac{1}{2},\frac{-1}{4}$	$2+2+2$	$6\cdot \eta^{-1/3}$

Table 18: Chiral matter fields in the T_4^0, T_4^+, and T_4^- sectors.

B. Anomalies

The anomalies associated with the non-Abelian gauge groups turn out to be

$$\mathrm{Tr}[(\mathrm{NonAbel.})^2\cdot Y] = \mathrm{Tr}[(\mathrm{NonAbel.})^2\cdot Q_6] = 0 \tag{B.1}$$

$$\begin{aligned}\mathrm{Tr}[(\mathrm{NonAbel.})^2\cdot Q_1] &= \mathrm{Tr}[(\mathrm{NonAbel.})^2\cdot Q_2] = \mathrm{Tr}[(\mathrm{NonAbel.})^2\cdot Q_3]\\ &= \mathrm{Tr}[(\mathrm{NonAbel.})^2\cdot Q_4] = -\frac{1}{2}\end{aligned} \tag{B.2}$$

$$\mathrm{Tr}[(\mathrm{NonAbel.})^2\cdot Q_{B-L}] = \mathrm{Tr}[(\mathrm{NonAbel.})^2\cdot Q_5] = +\frac{1}{2}, \tag{B.3}$$

$P+V$	χ	$(N^L)_j$	$\mathcal{P}_1(f_0)$	SM
$\left(\frac{1}{4},\frac{1}{4},\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{12},\frac{5}{12},\frac{1}{12}\right)\left(\frac{1}{4},\frac{3}{4};0^6\right)'$	L	1_3	1	$\mathbf{1_0}$
$\left(\frac{1}{4},\frac{1}{4},\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{12},\frac{5}{12},\frac{1}{12}\right)\left(\frac{-3}{4},\frac{-1}{4};0^6\right)'$	L	1_3	1	$\mathbf{1_0}$
$\left(\frac{-1}{4},\frac{-1}{4},\frac{-1}{4};\frac{-1}{4},\frac{-1}{4};\frac{-1}{12},\frac{-1}{12},\frac{-5}{12}\right)\left(\frac{1}{4},\frac{3}{4};0^6\right)'$	L	2_3	1	$\mathbf{1_0}$
$\left(\frac{-1}{4},\frac{-1}{4},\frac{-1}{4};\frac{-1}{4},\frac{-1}{4};\frac{-1}{12},\frac{-1}{12},\frac{-5}{12}\right)\left(\frac{-3}{4},\frac{-1}{4};0^6\right)'$	L	2_3	1	$\mathbf{1_0}$
$P+V_+$	χ	$(N^L)_j$	$\mathcal{P}_1(f_+)$	SM
$\left(\underline{\frac{-7}{12},\frac{5}{12},\frac{5}{12}};\frac{1}{12},\frac{1}{12};\frac{-5}{12},\frac{-1}{12},\frac{3}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	L	1_3	1	α^0
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\underline{\frac{7}{12},\frac{-5}{12}};\frac{1}{12},\frac{5}{12},\frac{-3}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	L	2_3	1	$\overline{\delta}^{-1/6}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{7}{12},\frac{7}{12};\frac{1}{12},\frac{-7}{12},\frac{-3}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	L	0	1	$\overline{\xi}^{-2/3}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{-5}{12},\frac{-5}{12};\frac{1}{12},\frac{5}{12},\frac{9}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	L	0	1	$\overline{\eta}^{1/3}$
$\left(\frac{5}{12},\frac{5}{12},\frac{5}{12};\frac{1}{12},\frac{1}{12};\frac{7}{12},\frac{-1}{12},\frac{3}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	L	1_3	1	$\overline{\eta}^{1/3}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{-5}{12},\frac{-5}{12};\frac{1}{12},\frac{-7}{12},\frac{-3}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	L	2_3	1	$\overline{\eta}^{1/3}$
$P+V_-$	χ	$(N^L)_j$	$\mathcal{P}_1(f_-)$	SM
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\underline{\frac{-7}{12},\frac{5}{12}};\frac{3}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{-9}{12},\frac{1}{12},\frac{4}{12};0^5\right)'$	L	1_3	1	$\delta^{1/6}$
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\underline{\frac{-7}{12},\frac{5}{12}};\frac{3}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{3}{12},\frac{1}{12},\frac{-8}{12};0^5\right)'$	L	2_3	1	$\delta^{1/6}$
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\frac{5}{12},\frac{5}{12};\frac{-9}{12},\frac{-1}{12},\frac{-1}{12}\right)\left(\frac{3}{12},\frac{1}{12},\frac{-8}{12};0^5\right)'$	L	0	1	$\eta^{-1/3}$

Table 19: Chiral matter fields satisfying $\Theta=0$ in the T_1^0 and $T_1^{\pm}$ sectors.

where NonAbel. = $SU(3)_c$, $SU(2)_L$, and $SO(10)'$. $U(1)^3$ type anomalies are

$$\mathrm{Tr}[(Q_Y)^3] = \mathrm{Tr}[(Q_Y)^2\cdot Q_6] = 0 \tag{B.4}$$

$$\begin{aligned}\mathrm{Tr}[(6Q_Y)^2\cdot Q_1] = \mathrm{Tr}[(6Q_Y)^2\cdot Q_2] &= \mathrm{Tr}[(6Q_Y)^2\cdot Q_3] \\ &= \mathrm{Tr}[(6Q_Y)^2\cdot Q_4] = -30 \end{aligned}\tag{B.5}$$

$$\mathrm{Tr}[(6Q_Y)^2\cdot Q_{B-L}] = \mathrm{Tr}[(6Q_Y)^2\cdot Q_5] = +30, \tag{B.6}$$

and

$$\mathrm{Tr}[(Q_6)^3] = \mathrm{Tr}[(Q_6)^2\cdot Q_Y] = 0 \tag{B.7}$$

$$\begin{aligned}\mathrm{Tr}[(Q_6)^2\cdot Q_1] &= \mathrm{Tr}[(Q_6)^2\cdot Q_2] = \mathrm{Tr}[(Q_6)^2\cdot Q_3] \\ &= \mathrm{Tr}[(Q_6)^2\cdot Q_4] = -4 \end{aligned}\tag{B.8}$$

$$\mathrm{Tr}[(Q_6)^2\cdot Q_{B-L}] = \mathrm{Tr}[(Q_6)^2\cdot Q_5] = +4, \tag{B.9}$$

and so on.

Thus, the anomaly free U(1) charge operators are Q_Y, Q_6, and

$$Q_a = Q_1 - Q_2, \tag{B.10}$$

$$Q_b = Q_1 + Q_2 - 2Q_3, \tag{B.11}$$

$$Q_c = Q_1 + Q_2 + Q_3 - 3Q_4, \tag{B.12}$$

$$Q_d = Q_1 + Q_2 + Q_3 + Q_4 + 4Q_5, \tag{B.13}$$

$$Q_e = Q_1 + Q_2 + Q_3 + Q_4 - Q_5 - \frac{1}{6}X. \tag{B.14}$$

$P+5V$	χ	$(N^L)_j$	$\mathcal{P}_5(f_0)$	SM
$\left(\frac{-1}{4},\frac{-1}{4},\frac{-1}{4},\frac{-1}{4},\frac{-1}{4};\frac{7}{12},\frac{-5}{12},\frac{-1}{12}\right)\left(\frac{1}{4},\frac{3}{4};0^6\right)'$	R	0	1	$\mathbf{1_0}^*$
$\left(\frac{-1}{4},\frac{-1}{4},\frac{-1}{4},\frac{-1}{4},\frac{-1}{4};\frac{7}{12},\frac{-5}{12},\frac{-1}{12}\right)\left(\frac{-3}{4},\frac{-1}{4};0^6\right)'$	R	0	1	$\mathbf{1_0}^*$
$\left(\frac{-1}{4},\frac{-1}{4},\frac{-1}{4},\frac{-1}{4},\frac{-1}{4};\frac{-5}{12},\frac{7}{12},\frac{-1}{12}\right)\left(\frac{1}{4},\frac{3}{4};0^6\right)'$	R	0	1	$\mathbf{1_0}^*$
$\left(\frac{-1}{4},\frac{-1}{4},\frac{-1}{4},\frac{-1}{4},\frac{-1}{4};\frac{-5}{12},\frac{7}{12},\frac{-1}{12}\right)\left(\frac{-3}{4},\frac{-1}{4};0^6\right)'$	R	0	1	$\mathbf{1_0}^*$
$\left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4};\frac{1}{12},\frac{1}{12},\frac{-7}{12}\right)\left(\frac{1}{4},\frac{3}{4};0^6\right)'$	R	1_1	1	$\mathbf{1_0}^*$
$\left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4};\frac{1}{12},\frac{1}{12},\frac{-7}{12}\right)\left(\frac{-3}{4},\frac{-1}{4};0^6\right)'$	R	1_1	1	$\mathbf{1_0}^*$
$P+5V_+$	χ	$(N^L)_j$	$\mathcal{P}_5(f_+)$	SM
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\underline{\frac{-7}{12},\frac{5}{12}};\frac{-1}{12},\frac{7}{12},\frac{3}{12}\right)\left(\frac{3}{12},\frac{1}{12},\frac{-8}{12};0^5\right)'$	R	0	1	$\overline{\delta}^{-1/6\ *}$
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\frac{-7}{12},\frac{-7}{12};\frac{-1}{12},\frac{-5}{12},\frac{3}{12}\right)\left(\frac{3}{12},\frac{1}{12},\frac{-8}{12};0^5\right)'$	R	0	1	$\overline{\xi}^{-2/3\ *}$
$\left(\frac{-5}{12},\frac{-5}{12},\frac{-5}{12};\frac{-1}{12},\frac{-1}{12};\frac{5}{12},\frac{1}{12},\frac{-3}{12}\right)\left(\frac{-9}{12},\frac{1}{12},\frac{4}{12};0^5\right)'$	R	0	1	$\overline{\eta}^{1/3\ *}$
$\left(\frac{-5}{12},\frac{-5}{12},\frac{-5}{12};\frac{-1}{12},\frac{-1}{12};\frac{5}{12},\frac{1}{12},\frac{-3}{12}\right)\left(\frac{3}{12},\frac{1}{12},\frac{-8}{12};0^5\right)'$	R	1_1	1	$\overline{\eta}^{1/3\ *}$
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\frac{5}{12},\frac{5}{12};\frac{-1}{12},\frac{-5}{12},\frac{3}{12}\right)\left(\frac{-9}{12},\frac{1}{12},\frac{4}{12};0^5\right)'$	R	1_1	1	$\overline{\eta}^{1/3\ *}$
$\left(\frac{1}{12},\frac{1}{12},\frac{1}{12};\frac{5}{12},\frac{5}{12};\frac{-1}{12},\frac{-5}{12},\frac{3}{12}\right)\left(\frac{3}{12},\frac{1}{12},\frac{-8}{12};0^5\right)'$	R	2_1	1	$\overline{\eta}^{1/3\ *}$
$P+5V_-$	χ	$(N^L)_j$	$\mathcal{P}_5(f_-)$	SM
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\underline{\frac{7}{12},\frac{-5}{12}};\frac{9}{12},\frac{1}{12},\frac{1}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	R	0	1	$\delta^{1/6\ *}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{7}{12},\frac{7}{12};\frac{-3}{12},\frac{1}{12},\frac{1}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	R	2_1	1	$\xi^{2/3\ *}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{-5}{12},\frac{-5}{12};\frac{-3}{12},\frac{1}{12},\frac{1}{12}\right)\left(\frac{-9}{12},\frac{-7}{12},\frac{-4}{12};0^5\right)'$	R	0	1	$\eta^{-1/3\ *}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{-5}{12},\frac{-5}{12};\frac{-3}{12},\frac{1}{12},\frac{1}{12}\right)\left(\frac{3}{12},\frac{-7}{12},\frac{8}{12};0^5\right)'$	R	1_1	1	$\eta^{-1/3\ *}$
$\left(\frac{5}{12},\frac{5}{12},\frac{5}{12};\frac{1}{12},\frac{1}{12};\frac{3}{12},\frac{-5}{12},\frac{-5}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	R	1_1	1	$\eta^{-1/3\ *}$
$\left(\frac{-1}{12},\frac{-1}{12},\frac{-1}{12};\frac{-5}{12},\frac{-5}{12};\frac{-3}{12},\frac{1}{12},\frac{1}{12}\right)\left(\frac{3}{12},\frac{5}{12},\frac{-4}{12};0^5\right)'$	R	$1_{\bar{2}},4_1$	$1+1$	$2\cdot\eta^{-1/3\ *}$

Table 20: Chiral matter fields satisfying $\Theta=0$ in the T_5^0 and $T_5^{\pm}$ sectors. They are all the right-handed states. Their $\mathcal{CTP}$ conjugates with the left-handed chirality are provided from the T_7^0, T_7^+, and T_7^- sectors.

The anomalous $\mathrm{U}(1)_\mathrm{A}$ is given by

$$Q_A = Q_1 + Q_2 + Q_3 + Q_4 - Q_5 + 6X. \tag{B.15}$$

It can be shown that the gravitational anomalies are $\mathrm{Tr}Q_Y = \mathrm{Tr}Q_6 = \mathrm{Tr}Q_a = \mathrm{Tr}Q_b = \mathrm{Tr}Q_c = \mathrm{Tr}Q_d = \mathrm{Tr}Q_e = 0$, and $\mathrm{Tr}Q_A = -50$. It can be cancelled via the Green-Schwarz mechanism [21].

References

[1] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Strings on orbifolds*, *Nucl. Phys.* **B 261** (1985) 678; *Strings on orbifolds. 2*, *Nucl. Phys.* **B 274** (1986) 285.

[2] L.E. Ibáñez, H.P. Nilles and F. Quevedo, *Orbifolds and Wilson lines*, *Phys. Lett.* **B 187** (1987) 25.

[3] K.-S. Choi and J. E. Kim, *Quarks and leptons from orbifolded superstring*, Springer-Verlag, Heidelberg, Germany (2006).

[4] L.E. Ibáñez, J.E. Kim, H.P. Nilles and F. Quevedo, *Orbifold compactifications with three families of* $SU(3) \times SU(2) \times U(1)^n$, *Phys. Lett.* **B 191** (1987) 282;
J.A. Casas and C. Muñoz, *Three generation* $SU(3) \times SU(2) \times U(1)_Y$ *models from orbifolds*, *Phys. Lett.* **B 214** (1988) 63.

[5] Fermionic constructions and Calabi-Yau space constructions toward standard-like models appear in the literature:
G.B. Cleaver, A.E. Faraggi and D.V. Nanopoulos, *String derived MSSM and M-theory unification*, *Phys. Lett.* **B 455** (1999) 135 [hep-ph/9811427];
A.E. Faraggi, E. Manno and C. Timirgaziu, *Minimal standard heterotic string models*, *Eur. Phys. J.* **C 50** (2007) 701 [hep-th/0610118];
G. Cleaver, M. Cvetič, J.R. Espinosa, L.L. Everett and P. Langacker, *Flat directions in three-generation free-fermionic string models*, *Nucl. Phys.* **B 545** (1999) 47 [hep-th/9805133];
V. Braun, Y.-H. He, B.A. Ovrut and T. Pantev, *The exact MSSM spectrum from string theory*, *JHEP* **05** (2006) 043 [hep-th/0512177];
V. Bouchard and R. Donagi, *An* $SU(5)$ *heterotic standard model*, *Phys. Lett.* **B 633** (2006) 783 [hep-th/0512149];
For intersecting branes, we list just a first few out of numerous attempts:
A. Font and L.E. Ibáñez, *SUSY-breaking soft terms in a MSSM magnetized D7-brane model*, *JHEP* **03** (2005) 040 [hep-th/0412150];
L.E. Ibáñez, *The fluxed MSSM*, *Phys. Rev.* **D 71** (2005) 055005 [hep-ph/0408064];
D. Lüst, S. Reffert and S. Stieberger, *MSSM with soft SUSY breaking terms from D7-branes with fluxes*, *Nucl. Phys.* **B 727** (2005) 264 [hep-th/0410074];
M. Cvetič, T. Li and T. Liu, *Standard-like models as type-IIB flux vacua*, *Phys. Rev.* **D 71** (2005) 106008 [hep-th/0501041].

[6] J.E. Kim, *Z(3) orbifold construction of* $SU(3)^3$ *GUT with* $\sin^2\theta_W = 3/8$, *Phys. Lett.* **B 564** (2003) 35 [hep-th/0301177].

[7] K.-S. Choi, K.Y. Choi, K.-w. Hwang and J.E. Kim, *Higgsino mass matrix ansatz for MSSM*, *Phys. Lett.* **B 579** (2004) 165 [hep-ph/0308160].

[8] J.E. Kim, *Trinification with* $\sin^2\theta_W = 3/8$ *and seesaw neutrino mass*, *Phys. Lett.* **B 591** (2004) 119 [hep-ph/0403196];
J. Giedt, G.L. Kane, P. Langacker and B.D. Nelson, *Massive neutrinos and (heterotic) string theory*, *Phys. Rev.* **D 71** (2005) 115013 [hep-th/0502032].

[9] T. Kobayashi, S. Raby and R.-J. Zhang, *Searching for realistic 4D string models with a Pati-Salam symmetry: orbifold grand unified theories from heterotic string compactification on a Z(6) orbifold*, *Nucl. Phys.* **B 704** (2005) 3 [hep-ph/0409098].

[10] W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, *Dual models of gauge unification in various dimensions*, *Nucl. Phys.* **B 712** (2005) 139 [hep-ph/0412318]; *Supersymmetric standard model from the heterotic string. II*, hep-th/0606187.

[11] J.E. Kim and B. Kyae, *String MSSM through flipped* $SU(5)$ *from* $Z(12-I)$ *orbifold*, hep-th/0608085; *Flipped* $SU(5)$ *from* $Z(12-I)$ *orbifold with wilson line*, *Nucl. Phys.* **B 770** (2007) 47 [hep-th/0608086].

[12] T. Huang and X.-G. Wu, *Determination of the* η *and* η' *mixing angle from the pseudoscalar transition form factors*, *Eur. Phys. J.* **C 50** (2007) 771 [hep-ph/0612007].

[13] I.-W. Kim, J.E. Kim and B. Kyae, *Harmless R-parity violation from Z_{12-I} compactification of $E_8 \times E_8'$ heterotic string*, *Phys. Lett.* **B 647** (2007) 275 [hep-ph/0612365].

[14] See, for example, L.E. Ibáñez and G.G. Ross, *Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model*, *Nucl. Phys.* **B 368** (1992) 3;
B.C. Allanach, A. Dedes and H.K. Dreiner, *Bounds on R-parity violating couplings at the weak scale and at the GUT scale*, *Phys. Rev.* **D 60** (1999) 075014 [hep-ph/9906209].

[15] D.J. Gross, J.A. Harvey, E.J. Martinec and R. Rohm, *The heterotic string*, *Phys. Rev. Lett.* **54** (1985) 502.

[16] L.M. Krauss and F. Wilczek, *Discrete gauge symmetry in continuum theories*, *Phys. Rev. Lett.* **62** (1989) 1221;
T. Banks, *Effective lagrangian description of discrete gauge symmetries*, *Nucl. Phys.* **B 323** (1989) 90.

[17] S. Förste, H.P. Nilles, P.K.S. Vaudrevange and A. Wingerter, *Heterotic brane world*, *Phys. Rev.* **D 70** (2004) 106008 [hep-th/0406208];
S. Förste, T. Kobayashi, H. Ohki and K.-j. Takahashi, *Non-factorisable $Z_2 \times Z_2$ heterotic orbifold models and Yukawa couplings*, *JHEP* **03** (2007) 011 [hep-th/0612044].

[18] P.H. Ginsparg, *Gauge and gravitational couplings in four-dimensional string theories*, *Phys. Lett.* **B 197** (1987) 139.

[19] O. Lebedev et al., *A mini-landscape of exact MSSM spectra in heterotic orbifolds*, *Phys. Lett.* **B 645** (2007) 88 [hep-th/0611095].

[20] M. Dine, N. Seiberg and E. Witten, *Fayet-Iliopoulos terms in string theory*, *Nucl. Phys.* **B 289** (1987) 589;
J.J. Atick, L.J. Dixon and A. Sen, *String calculation of Fayet-Iliopoulos D terms in arbitrary supersymmetric compactifications*, *Nucl. Phys.* **B 292** (1987) 109;
M. Dine, I. Ichinose and N. Seiberg, *F terms and D terms in string theory*, *Nucl. Phys.* **B 293** (1987) 253.

[21] M.B. Green and J.H. Schwarz, *Anomaly cancellation in supersymmetric d=10 gauge theory and superstring theory*, *Phys. Lett.* **B 149** (1984) 117.

[22] N. Sakai and T. Yanagida, *Proton decay in a class of supersymmetric grand unified models*, *Nucl. Phys.* **B 197** (1982) 533;
S. Weinberg, *Supersymmetry at ordinary energies. 1. Masses and conservation laws*, *Phys. Rev.* **D 26** (1982) 287.

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